

S

THE
PRACTICAL GAGER:
OR, THE
YOUNG GAGER'S ASSISTANT.

CONTAINING

Those Things which are actually practised, and which are absolutely necessary to be known and understood by every Person that is employed as a Gager or Officer in the Revenue of Excise.

To which are added,

All the necessary Tables for gaging and fixing ~~the~~ Utensils of Victuallers, Common Brewers, and Distillers: Also for moneying the several sorts of Goods, or for finding the Amounts of the Charges.

Very useful for SUPERVISORS, OFFICERS, and COLLECTORS CLERKS.

A NEW EDITION.

With an APPENDIX;

Containing several Additions, viz. The Method of Gaging by equidistant Ordinates; Inching and Tabulating close Casks; and Specimens of Vouchers and Abstracts for the several Duties.

Bonum quò communius eò melius.

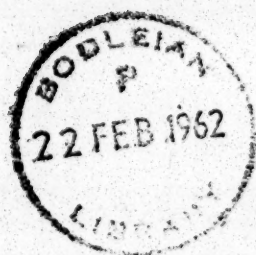
Dedicated (by Permission) to the Honourable Commissioners of Excise.

By WILLIAM SYMONS,
COLLECTOR OF EXCISE.

L O N D O N:

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TO THE
HONOURABLE
COMMISSIONERS
AND
GOVERNORS
OF HIS MAJESTY'S
REVENUE OF EXCISE, &c.

HONOURABLE SIRS,

THE ensuing treatise was compiled chiefly for the assistance and use of the Officers employed in his Majesty's revenue of Excise, &c. under your Honours management and direction; which being the product of my leisure hours in that station wherein you were pleased to place me, it naturally and of right belongs to you; therefore (as in duty bound) I humbly beg leave to address

it to your patronage and protection. And if it shall meet with your Honours approbation, and prove of general utility to those persons for whom it was intended, the compiler will have his end answered, who is,

Honoured Sirs,

Your most dutiful

and obedient Servant,

WILLIAM SYMONS.

P R E F A C E.

IN the following pages I have collected together all the most useful and necessary directions that are practised by the officers of the excise, in the whole art of gaging, and given the most concise rules for performing them. Throughout the whole I have endeavoured to free the art from all those obscurities, and mixtures of other things, which are frequently met with in books of gaging, are foreign to the purpose, and of no service to the practical gager, but to puzzle and perplex, rather than to inform him.

I have proceeded gradually from step to step in every respect, reduced the theory of the art to the present method of practice, and have laid down the whole in the plainest and easiest manner, by giving familiar rules and examples, wrought both by pen and rule; so that any one but of an ordinary capacity may readily comprehend them, and soon become master of the whole art.

The methods of performing the business of gaging are not only here shewn, but the grounds and reasons also why the gager is to proceed in such and such a manner; which will enable him to understand what he is about when he is upon his duty. On those heads where others have dwelt too long, I have been the shorter: and where they have been too brief, those heads I have enlarged and explained.

In this undertaking I have strictly adhered to those methods which are now made use of by the officers of excise; and though many of the examples are wrought by a different process, yet the solutions are exactly the same; so that the young gager is left to his choice in the use of that operation which he finds to be fittest for his purpose.

I am sensible that there have already been a multitude of books written on this subject by persons of superior abilities; many of whom have treated it in a very learned and judicious manner, as far as it relates to the theory of the art. And others too have handled the

practical part of gaging very well; that at first view one would be ready to imagine that there could be nothing further needful written on this head. But on a closer review of the several treatises extant on this subject, one does not find any of them so well adapted to the present method of practice as one might reasonably expect; for some of them are written in a style beyond the reach of a common capacity to comprehend, unless he were acquainted with the algebraic art; and others who have treated on the practical parts of gaging, have entirely neglected, or superficially passed over, several useful things which are of absolute necessity in practice.

On this account, and at the instigation of some of my friends in the excise, I was prevailed upon to compile this tract, which, without any farther apology, I here present the reader with, relying upon his candour in excusing the inaccuracy of style, or any other errors I may have been guilty of in the course of it.

I doubt not but many into whose hands this book may come, will be ready to object, "that they can see nothing in it but what every officer of excise must know;" and this I as readily acknowledge. But the design of its publication is not for the information of such as know these things already, but for the sake of young beginners, and candidates for the excise. Besides, it would be a sign of presumption in me to attempt to introduce new methods of gaging into practice, when those already made use of, and which are enjoined by my superiors, so fully answer the end to all intents and purposes.

My intent was only to collect together, in as small a compass as possible, every thing that would be useful, necessary, or curious in the art of gaging; and to lay down rules and precedents to be applied upon every emergency without the help of any other book.

How far I have answered this end is submitted to the practical gager. And if what I have done shall prove of any service to my brother officers, I shall think my labour and pains very well bestowed.

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EXPLANATION of the CHARACTERS.

FOR the sake of shortening the work I have made use of a few algebraic signs or characters, which are explained as follows.

Signs or
characters.

+ more. { This character is the sign of addition :
As $6 + 5$ signifies that 6 and 5 are
to be added together.

— less. { This character is the sign of subtraction : As $9 - 4$ signifies that 4 is to
be subtracted from 9.

x multiplied. { This character is a sign of multiplication : As 7×3 signifies that 7 is
to be multiplied by 3.

= equal. { This character is a sign of equality :
As $7 = 7$, or $6 + 5 = 11$, or $12 - 3 = 9$ signifies that 7 is equal to 7 ;
6 more 5 is equal to 11 ; and 12
less 3 is equal to 9.

: : so is. { This character is a sign of proportion,
and is always put between the two
middle terms in the rule of three :

is to so is to

Thus as $4 : 8 :: 5 : 10$, which signifies that as 4 is to 8, so is 5 to 10.

X Strong. This is the character of strong beer.

VI Small. This is the character of small.

A. G.	} signifies {	Ale gallons.
W. G.		Wine gallons.
M. B.		Malt bushels.
G. P.		The gage point.
S. G. P.		The square gage point.
C. G. P.		The circular gage point.

THE PRACTICAL GAGER.

CHAP. I.

Of DECIMAL FRACTIONS.

MY design is not here to treat the subject of Decimal Fractions at large, for that would require a volume of itself larger than I intend this to be; besides, that has been already done by so many hands, that I think it quite needless to add a great deal on that head. Therefore I shall only give a few short and easy rules, and subjoin some familiar examples, to assist the memory of those who have already learned it, which, if thoroughly understood, may be made applicable to all the various cases of practical gaging.

ART. I.

Of NOTATION of DECIMALS.

AS in whole numbers the value of every figure from the unit's place to the left hand increases in value, in a decuple or tenfold proportion; so in decimals every figure to the right hand of unity decreases in value in a decuple or tenfold proportion, as will appear by the following table.

B

Whole

Of Decimal Fractions.

Whole Numbers. Decimals.

XCM.	CM.	XM.	M.	C.	X.	Units.	X Parts.	C Parts.	M Parts.	XM Parts.	CM Parts.	XCM Parts.
7	6	5	4	3	2	1	2	3	4	5	6	7

And whereas cyphers at the left hand or before whole numbers signify nothing, as for example, 00365 = 365, and 0456 = 456; so cyphers at the right hand or after decimals signify nothing, as, ,36500 is the same as ,365, and ,4560 is = ,456. But cyphers at the left hand of decimals decrease their value in a tenfold proportion, as will appear by the following table.

Thus,			
,5	} is {	Tenth Parts	} of Unity
,05		C Parts	
,005		M Parts	
,0005		XM Parts	

ART. II.

Of ADDITION and SUBTRACTION of DECIMALS.

THIS is performed in the same manner as in common arithmetic, only remember to place primes under primes, seconds under seconds, thirds under thirds, &c. as in the following examples :

EXAMPLES.

ADDITION.

To	45,6053
Add	34,0632
Sum =	<u>79,6 85</u>

SUBTRACTION.

From	23,05640
Subtract	22,98764
Diff. =	<u>,06876</u>

To

To ,006532
Add ,765039

From 6,540938
Subtract 5,847603

Sum = ,771571

Diff. = ,693335

This being so easy that I shall add no more examples.

ART. III.

MULTIPLICATION of DECIMALS.

RULE I.

Multiply both factors together as in common arithmetic; when done, count how many decimals there were in both factors, and so many figures prick off in the product for decimals.

II. If it should happen that there should not be so many decimals in the product, as there were in the two factors, then add as many cyphers to the left hand of the product as will make up that deficiency. See the following examples:

EXAMPLES.

Multiply 8,506
By 6,296

51036
70554
17012
51036

Product 53,553776

,007654
,003542

15308
30616
38270
22962

,000027110468

In the first example there are three decimals in each factor: therefore I prick off six figures in the product for decimals; and in the second example there are six decimals in each factor, and their sum is equal to twelve: but there being only eight figures in the product, therefore I add four cyphers more to the left hand of the product, which makes the number twelve, equal to the

B 2

number

number of both factors, which is true. The like must be done in all cases.

ART. IV.

DIVISION of DECIMALS.

RULE I.

AS many more decimals as there are in the dividend than the divisor, so many figures prick off in the quotient for decimals.

II. But if there are not so many decimals in the dividend as there are in the divisor, then add as many cyphers to the right hand of the dividend as will make their number equal, and in that case the quotient will be integers, or whole numbers. See the following examples :

EXAMPLES.

$$282,).785400(.002785$$

$$\underline{564}$$

$$2214$$

$$\underline{1974}$$

$$2400$$

$$\underline{2256}$$

$$1440$$

$$\underline{1410}$$

$$30 \text{ Rem.}$$

$$,7854)282,0000(359,05$$

$$\underline{23562}$$

$$46380$$

$$\underline{39270}$$

$$71100$$

$$\underline{70686}$$

$$41400$$

$$\underline{39270}$$

$$2130$$

In the first example there are six decimals in the dividend, and none in the divisor; consequently six figures are to be pricked off for decimals in the quotient. But there are only four figures, *viz.* 2785, produced in the quotient by division; therefore I add two cyphers to the left hand of the quotient, which will make it true.

true. In the second example there are four decimals in the divisor, and none in the dividend, therefore I add four cyphers to the dividend, which makes their number equal, and the quotient comes out whole numbers, viz. 359. The other two figures are gained by adding cyphers to the remainder.

More EXAMPLES.

$$\begin{array}{r}
 25,6),5065(,019 \\
 \underline{256} \\
 2505 \\
 \underline{2304} \\
 201
 \end{array}$$

$$\begin{array}{r}
 ,005)725,000(145000 \\
 \underline{5} \\
 22 \\
 \underline{20} \\
 25 \\
 \underline{25}
 \end{array}$$

In the first of these examples there are three decimals more in the dividend than in the divisor; for which reason there must be three decimals in the quotient: Therefore to the quotient 19 I prefix a cypher, which makes it true.

In the second example the divisor consists of three places of decimals; and as there are no decimals in the dividend, I add three cyphers, which make their numbers equal, and the quotient comes out whole numbers.

And thus, by observing these general rules, the value of the quotient may be discovered in all cases.

A R T. V.

Of REDUCTION of DECIMALS.

TO reduce a vulgar fraction to its equivalent value in decimals.

R U L E.

Divide the numerator of the given vulgar fraction by its dominator, and the quotient will be the decimal required.

Of Decimal Fractions.

E X A M P L E S.

What are the decimals of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$?

O P E R A T I O N.

$$\begin{array}{r} 4 \overline{)1,00(.25} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{)10(.5} \\ \underline{10} \\ 0 \end{array}$$

$$\begin{array}{r} 4 \overline{)3,00(.75} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Thus I find the decimal of $\frac{1}{4} = ,25$, $\frac{1}{2} = ,5$, and $\frac{3}{4} = ,75$. In like manner may any other vulgar fraction be reduced when the numerator and denominator consists of more than one figure.

As for example, What is the Decimal of $\frac{17}{25}$?

O P E R A T I O N.

25 $\overline{)17,0(.68}$ = Decimal required.

$$\begin{array}{r} 150 \\ \underline{} \\ 200 \\ \underline{} \\ 200 \\ \underline{} \\ 0 \end{array}$$

Note, When there is a remainder, add cyphers to it? and continue the division till you have gained six places of decimals in the quotient, which will be sufficient for most purposes.

A R T. VI.

TO reduce the different denominations of money, weights, and measures, to their equivalent decimal expressions.

RULE

R U L E.

I. Place the several denominations orderly over one another, (the least uppermost) then,

II. Divide the lesser denomination by the number of units of that denomination contained in the next greater denomination, and add the quotient to the next greater denomination.

III. Divide this number by the number of units of this denomination contained in the next greater, and add the quotient to the next greater denomination, and so proceed to do till you arrive at the decimal of the integer, which will appear more plain by the following examples :

E X A M P L E S.

Let it be required to reduce 10s. 9d. and 15s. 6d. 3f. each to its equivalent decimal of a pound sterling.

O P E R A T I O N.

Pence 12|9
Shillings 20|10,75
 ,5375

Farthings 4|3
Pence 12|6,75
Shillings 20|15,5625
 ,778125

By which I find the decimal of 10s. and 9d. = ,5375 $\frac{1}{2}$. and that of 15s. 6d. 3f. = ,778125. And in like manner may any other denomination (whether they be weights or measures) be reduced to their equivalent decimal expressions.

A R T. VII.

TO reduce or find the value of any decimal fraction.

R U L E.

Multiply the given decimal by the number of units of the next lesser denomination contained in one of that

denomination which the decimal respects for its integer, and from that product cut off so many figures as there were in the given decimal, and this decimal multiply by the next lesser denomination, and prick off the first number of decimals, and thus proceed to do till you have multiplied by all the denominations contained in the integer. Then the whole numbers on the left hand of the separating point will be the value of the decimal required.

For example, What is the value of ,5375^l. and ,778125^l. ?

OPERATION.

,5375	,778125
20	20
10,7500	15,562500
12	12
9,0000	6,750000
	4
	3,000000

By which I find the value of ,5375 = 10s. 9d. and that of ,778125 = 15s. 6d. 3f. which are the same as were required to be reduced in the last article ; and in this manner may any other decimal fraction be reduced. But the value of the decimal of a pound sterling may be discovered by inspection only.

RULE.

The primes place double; and account so many shillings ; if the second figure be five or more, add one shilling more to the double of the first figure. And if any thing remain above five in the place of seconds, or if the seconds are under five, account them as so many tens, to which said tens add the figure in the third place, which will be so many units. Their sum account

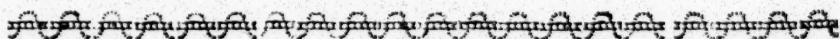
count as one entire number, which being made less by one in every twenty-five, will be farthings, to be added to the shillings before found, and this sum will be equal to the value of the given decimal, which will be more intelligible by the following examples :

E X A M P L E S.

What is the value of ,9625 $\frac{1}{2}$?

The first figure is 9, which doubled is 18; the second being 6, I take 5 from it, and add 1 to the 18, which makes 19s. The remainder of the second figure above 5 is one, which is 10 farthings, and the figure in the third place being 2, I add to the 10, which makes 12 farthings. The figure in the fourth place being 5, or a half farthing, is abated according to the rule; for if 25 abate 1, 12, 5 will abate 5 tenths. So I find the value of the given decimal to be equal to 19s. 3d.

In like manner may the value of the decimal ,5375 $\frac{1}{2}$ be found equal to 10s. 9d. And that of the decimal ,778125 $\frac{1}{2}$ equal to 15s. 6d. 3f. or the value of any other decimal of a pound sterling.



C H A P. II.

Of EXTRACTION *of the* SQUARE ROOT.

TO extract the Square Root of any whole or mixed number.

D E F I N I T I O N.

A square number is produced by multiplying any number by itself, as $4 \times 4 = 16$, the square of 4, and $5 \times 5 = 25$, the square of 5, &c.

B 5

The

Extraction of the Square Root.

The roots are either simple or compound. The simple roots with their squares are exhibited in the following table, which ought to be got by heart :

A Table of Roots and Squares.

Rs.	1	2	3	4	5	6	7	8	9
Sqs.	1	4	9	16	25	36	49	64	81

Compound roots are those which consist of more than one figure, as $36 \times$ by $36 = 1296$, the square of 36, &c. Therefore to extract the square root of any number is to find a number that being multiplied by itself will produce the number given. And in order to this observe the following rules.

I. Put a point over every second figure (beginning at the unit's place from the right hand to the left) ; and as many points or periods as there are in the given number, so many figures will be in the root.

II. See what is the nearest less square number in your first period to the left hand, which may be known by the foregoing table of roots and squares, and whatever you find it to be, place the root of it in the quotient, and the square of it under the first period, and subtract it from that period ; then to the remainder bring down the next two figures or period.

III. To find what the next figure of the root will be, double the root before found, and set it to the left hand of the remainder for a divisor ; then see how often you can have that divisor in the remainder, augmented by one of the figures last brought down ; and as often as you find it will go, place it in the root, and also in the unit's place of the divisor.

IV. Lastly, multiply this augmented divisor by the last figure set in the root, and set the product under the remainder, and subtract it from it, then bring down the figures in the next period, and add them to this last remainder. Then again double the root for a divisor, and thus proceed to do till all the periods are brought down ; and if any thing remains, add two cyphers to the remainder

remainder, and repeat the work. Note, for every pair of cyphers you add, you will have one decimal in the root. When the given number consists of whole numbers and decimals, the whole numbers must be pointed as before directed. But the decimals must be pointed the contrary way, *viz.* every second figure to the right hand from the unit's place; and when there is an odd decimal, add a cypher to make their number even.

E X A M P L E S.

What is the Square Root of 256?

O P E R A T I O N.

$$\begin{array}{r}
 \dot{2}5\dot{6}(16=\text{Root.} \\
 \text{1} \\
 \hline
 26)156 \\
 \underline{156} \\
 0
 \end{array}$$

In the above example there are two points, the first on the figure 2. The nearest less square to 2 is 1, which also is the root; therefore I place the 1 in the root, and also under the figure 2, which is the first period; then I subtract the figure 1 from 2, and there remains 1, to which I bring down the next period 56; then I double the root, *viz.* $1 \times 2 = 2$, which I set in the divisor's place; then I seek how often 2 will go in 15, which I find to be six times, therefore I place 6 in the root, and also in the divisor's place. Then I multiply my divisor 26 by 6, the last figure in the root, and the product 156 I subtract from the remainder augmented by the last period, and there remains nothing. So I find the root of 256 to be 16; for $16 \times 16 = 256$.

The way to prove the work is to multiply the root by itself, and the product, with the remainder (if any,) will be equal to the given square number, if right, otherwise not.

Extraction of the Square Root.

EXAMPLE II.

What is the Square Root of 282?

OPERATION.

$$\begin{array}{r}
 \dot{2}8\dot{2}(16,79 = \text{Root.} \\
 \underline{1} \\
 26)182 \\
 \underline{156} \\
 327)2600 \\
 \underline{2289} \\
 3349)31100 \\
 \underline{30141} \\
 \text{Remain. } 959
 \end{array}$$

EXAMPLE III.

What is the Square Root of 62650,9?

OPERATION.

$$\begin{array}{r}
 \dot{6}2\dot{6}5\dot{0},9\dot{0}(250,301 = \text{Root.} \\
 \underline{4} \\
 45)226 \\
 \underline{225} \\
 5003)15090 \\
 \underline{15009} \\
 500601)810000 \\
 \underline{500601} \\
 309399 \text{ Remainder.}
 \end{array}$$

When you cannot have your divisor in your dividend (without counting the figure in the unit's place,) then

then add a cypher to the root, and also to the divisor, as in the last example, and bring down the next period (if any,) or add a pair of cyphers, and proceed according to the foregoing rules. And in this manner may the square root of any other whole or mixed number be extracted.

II. *To extract the SQUARE ROOT of a DECIMAL FRACTION.*

THIS differs nothing from the extracting the roots of whole numbers, except in pricking off the roots; for as in whole numbers we begin to prick off every second figure from the right hand to the left, so in decimals we must begin at the left hand, and prick off every second figure to the right hand, which is just the reverse.

Note, if the number of figures are odd, always add a cypher to the right hand of the given sum before you begin your work, to make them even. A few examples will make this plain.

EXAMPLE I.

What is the Square Root of ,00046225?

OPERATION.

,00046225 (,0215 = Root.

$$\begin{array}{r}
 4 \\
 \hline
 41)062 \\
 41 \\
 \hline
 425)2125 \\
 2125 \\
 \hline

 \end{array}$$

o Remainder.

E X A M-

Extraction of the Square Root.

EXAMPLE II.

What is the Square Root of ,06435?

OPERATION.

,064350(,25367 = Root.

$$\begin{array}{r}
 4 \\
 \hline
 45 \overline{) 243} \\
 \underline{225} \\
 503 \overline{) 1850} \\
 \underline{1509} \\
 5066 \overline{) 34100} \\
 \underline{30396} \\
 50727 \overline{) 370400} \\
 \underline{355089} \\
 15311 \text{ Remainder.}
 \end{array}$$

EXAMPLE III.

What is the Square Root of ,35864?

OPERATION.

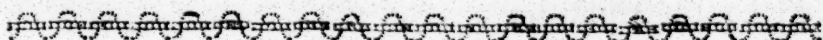
,358640(,598865 = Root.

$$\begin{array}{r}
 25 \\
 \hline
 109 \overline{) 1086} \\
 \underline{981} \\
 1188 \overline{) 10540} \\
 \underline{9504} \\
 11968 \overline{) 103600} \\
 \underline{95744} \\
 119766 \overline{) 785600} \\
 \underline{718596} \\
 1197725 \overline{) 6700400} \\
 \underline{5988625} \\
 711775 \text{ Remainder.}
 \end{array}$$

Thus

Thus may the roots be found to what exactness you please by constantly adding two cyphers to the remainder. But six decimals are enough in most cases, and even two are enough in many cases.

If what has been said on this head be perfectly understood, it will be no difficult matter to extract the square root of any number whatsoever; therefore I shall add no more examples on this head, but proceed to the next.



CHAP. III.

Of EXTRACTION of the CUBE ROOT.

DEFINITION.

I. **A** Cube number is produced by multiplying any number by itself, and again multiplying that product by the same number: That is, if you square any number, and multiply that square by its root, the product will be the cube of that number. Thus, $2 \times 2 = 4$, the square of 2: and $4 \times 2 = 8$, the cube of 2, Again, $5 \times 5 = 25$, the square of 5: and $25 \times 5 = 125$, the cube of 5; and so of any other.

II. The roots are either simple or compound. The simple roots are found by inspection as in the following table:

A Table of Roots and Cubes.

Rs.	1	2	3	4	5	6	7	8	9
Cs.	1	8	27	64	125	216	343	512	729

The compound roots are such as consist of two or more figures; as 12 is a compound root, and $12 \times 12 = 144$, the square of 12; and $144 \times 12 = 1728$, the cube of 12; and so of any other number.

To extract the COMPOUND CUBE ROOT of any number given.
R U L E.

I. **A**S in the extraction of the square root, put a point over the unit's place of the given number, and instead of putting the next points over every second figure, put them over every third figure towards the left hand; and as many points or periods as there are, so many figures will be in the root. If there are any decimals in the given number, then prick off every third decimal from the unit's place the contrary way, *viz.* from the left to the right hand; and when there happens to be but one or two decimals in the given number, then add one or two cyphers to make up three figures in order to compleat the period.

II. Find the nearest lesser cube (by the foregoing table) in the first period towards the left hand, and place the root of it on the right side of the crooked line, and the cube of it, set underneath the first period, and subtract it from it, and to the remainder bring down the next period, which call the resolvend.

III. To find the next figure in the root, triple the root, and set the unit's place of the triple root under the unit's place of the resolvend. Then square the root, and triple that square, the product of which set down as in multiplication under the triple root, *viz.* units under the ten's place; then draw a line, and add these two numbers together for a divisor.

IV. Find how often you can have your divisor in the resolvend, without counting the unit's place of the resolvend. And as often as you can have your divisor in that part of the resolvend set in the root's place, and observe the following directions:

1. Cube the figure last placed in the root, and set the unit's place of that cube under the unit's place of the resolvend.

2. Square the same, and multiply that square by the triple root before found, and set the unit's place of this product under the ten's place of the cube.

3. Mul-

3. Multiply the triple square of the root by the last figure placed in the root, which product set down under the product of the square and triple root, placing units under tens as before.

4. Draw a line, and add these three sums together, and subtract it from the resolvend, and to the remainder bring down the next period (if any,) or add three cyphers, which will be a new resolvend; to which, by the foregoing rules, find a new divisor, and thus proceed in all the steps as before directed, till all the periods are brought down, or till you have gained as many decimals in the root as you shall find needful. The following examples will make it more plain.

EXAMPLE I.

What is the Cube Root of 1728?

OPERATION.

$$\begin{array}{r}
 \overset{\cdot}{1}728 \text{ (12 = Root.} \\
 \underline{1} \quad \text{Cube of 1 subtract.} \\
 728 \text{ Resolvend.} \\
 \underline{3} \quad \text{Triple of the Root 1.} \\
 \underline{3.} \quad \text{Triple Square of the Root 1.} \\
 \underline{33} \quad \text{Divisor.} \\
 \underline{8} \quad \text{Cube of 2.} \\
 12 \quad \text{Triple of Root 1 by Square of 2.} \\
 \underline{6} \quad \text{Triple Square Root 1 by 2.} \\
 \text{Sum } \underline{728} \quad \text{Subtract from Resolvend.} \\
 0 \quad \text{Remain.}
 \end{array}$$

In the foregoing example I put a point over the figure 8 in the unit's place, and on the figure 1, which is the third figure towards the left hand; then the first period is 1, and the nearest less cube of 1 is 1, and 1 is also the root, which I place in the root's place, and under the first period; then 1 from 1 subtracted, leaves nothing, to which I bring down the next period 728, which is the resolvend. The root 1 tripled is 3, which I place under the units place of the resolvend, and the square of 1 is 1, which tripled is also 3, which I place

in

in the ten's place under the triple root. The sum of these two numbers is 33, which is the divisor: Then I seek how often I can have 33 in 72, the two first figures of my resolvend, which I find to be twice: Therefore I set the figure 2 in the root's place, then I cube the figure 2, which is equal to 8, and set it under the unit's place of the resolvend; then the square of $2=4$ I multiply by the triple root 3, which $=12$, and set it down under the ten's place of the cube; and lastly, I multiply the triple square of the root 1 by 2, which $=6$, and place it under the ten's place of the last product. The sum of these three numbers thus found $=728$, I subtract from the resolvend, and I find nothing remains, and the root required $=12$.

EXAMPLE II.

What is the Cube Root of 34645976?

OPERATION.

34645976	(326 = Root.
<u>27</u>	Cube of the Root 3.
7645	Resolvend.
<u>9</u>	Triple of the Root 3.
27	Triple Square of the Root 3.
<u>279</u>	Divisor.
8	Cube of 2.
36	Square of 2 $\times$ by the Triple Root.
54	Triple Square of the Root $\times$ by 2.
Sum 5768	Subtract from the Resolvend.
<u>1877976</u>	Resolvend.
96	Triple Root 32.
3072	Triple Square of the Root 32.
<u>30816</u>	Divisor.
216	Cube of 6.
3456	Square of 6 $\times$ into the Triple Root.
18432	Triple Square of the Root $\times$ by 6.
Sum 1877976	Subtract from last Resolvend.
Remains 0	

EXAM-

EXAMPLE III.

What is the Cube Root of 47,5 ?

OPERATION.

47,500(3,62 = Root.

$$\begin{array}{r}
 27 \\
 \hline
 20500 \text{ Resolvend.} \\
 9 \\
 27 \\
 \hline
 279 \text{ Divisor.} \\
 216 \\
 324 \\
 162 \\
 \hline
 19656 \text{ Subtrahend.} \\
 844000 \text{ Resolvend.} \\
 108 \\
 3888 \\
 \hline
 38988 \text{ Divisor.} \\
 8 \\
 432 \\
 7776 \\
 \hline
 781928 \text{ Subtrahend.} \\
 \text{Rem. } 62072
 \end{array}$$

Thus I find the root of $47,5 = 3,62$, and something more. If you have a mind to have the root nearer the truth, you may, by adding three cyphers to the remainder for every decimal you gain in the root, come to what exactness you please. Here observe, that it will be necessary to square the root upon a piece of waste paper when it becomes large, and also to find your subtrahend in the same manner, before you enter it in its proper place; for then, if you find it to exceed your particular resolvend, you may place a less figure in the root, and work again afresh for a new subtrahend, which must not exceed the resolvend.

The

The way to prove your work is to multiply the root by itself, and that product by the root; which sum, with the remainder, will be equal to the given whole number if right, otherwise not.

II. *Of the Use of the CUBE Root.*

THE chief use of the cube root is to discover the proportion of lines to solids, and of solids to lines; for like solids, such as globes, cylinders, cubes, &c. are in triplicate proportion to their like solids; therefore, as the cube of the diameter of any solid is to its weight or solidity, so is the cube of the diameter of another like solid to the weight or solidity thereof.

EXAMPLE I.

If an iron bullet of four inches diameter, weigh 9 pounds, what will an iron bullet of 8 inches diameter weigh?

RULE.

Multiply the cube of the third term by the second term, and divide the product by the cube of the first term, and the quotient will be the answer.

OPERATION.

Cube. Wt. Cube. Wt.
As 64 : 9 : 512 : 72 = Answer.

$$\begin{array}{r}
 9 \\
 \hline
 64 \overline{) 4608} \\
 \underline{448} \\
 128 \\
 \underline{128} \\
 0
 \end{array}$$

EXAM-

EXAMPLE II.

If a fathom of rope of 10 inches circumference weigh 17 lb. how much will a fathom weigh of 8 inches circumference?

OPERATION.

Cube of 10. lb. C. of 8. lb.

As 1000 : 17 : : 512 : $8\frac{704}{1000}$ = Answer.

$$\begin{array}{r} 17 \\ \hline 3584 \\ 512 \\ \hline \end{array}$$

1,000)8,704 = Remainder.

Another EXAMPLE.

Having the dimensions of a conical, or other vessel given with the content thereof, and I wanted to make another, that should contain more or less gallons, of a similar form.

RULE.

To solve such like questions, there must be three proportions wrought: I. As the content of the given vessel is to the cube of the given diameter at the top, so is the content of the vessel sought to the cube of its top diameter.

II. As the content of the given vessel is to the cube of its bottom diameter, so is the content of that sought to the cube of the required diameter.

III. As the content of the given vessel is to the cube of its depth, so is the content of the vessel sought to the cube of the depth sought. Then if you extract the cube root from these three numbers thus found, you will have the several dimensions required.

For EXAMPLE.

Suppose a conical tun of the following dimension and content, viz. top diameter = 30 inches, bottom diameter

diameter = 40 inches, depth = 20 inches, and the content = 68,7 ale gallons, and it were required to make another of the same form, that would contain 90 gallons, what must its dimensions be?

1. Proportion for the Top Diameter.

Cont.	Cube of T. Diameter.	Cont.	Cube of T. Diameter.
As 68,7	: 27000	: : 90	: 35371,179.

2. Proportion for the Bottom Diameter.

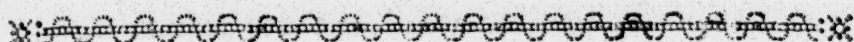
Gallons.	Cube of B. Diameter.	Gallons.	Cube of B. Diameter.
As 68,7	: 64000	: : 90	: 83842,795.

3. Proportion for the Depth.

Gallons.	Cube of Depth.	Gall.	Cube of Depth.
As 68,7	: 8000	: : 90	: 10480,349.

By extracting the cube roots of the fourth term of each of the three foregoing proportions, I find the top diameter required = 32,8 inches; the bottom diameter = 43,8, and the depth = 21,9 inches, the dimensions required.

But such kind of questions as these are solved with a vast deal less trouble by those sliding rules which have the line E upon them; for at once setting of the rule for each proportion, all the required dimensions are found without any more trouble. On account of the excellency of the cube line upon the rule for working any question in triplicate proportion, I have given rules for extracting the cube root by the pen in this book, in order to enable the young Gager to do it the better by the rule.



C H A P. IV.

A DESCRIPTION of the SLIDING RULE, and of the several lines upon it, with directions how to find any number thereon; together with the application of those lines to Multiplication, Division, the Rule of Three, &c.

A R T. I.

A DESCRIPTION of the RULE.

THIS instrument is commonly made of box or ivory, of divers lengths, viz. 6, 9, 12, 18 inches long, and sometimes longer; but that is not material, because they are all made after one and the same manner, with this only difference, the longer the lines are, the more subdivisions they will admit of.

What I shall here treat of is the common rule of 12 inches long, which, if thoroughly understood, that of any other length will be easily known.

An EXPLANATION of the LINES on the RULE.

I. **O**F the lines on the first face or side of the rule: The first line on this face is that on the rule which is distinguished by the letter **A**, and is commonly known by the name of *Gunter's Line*, it being a line of numbers in a geometrical proportion, consisting of two radiusses, and is numbered from the left to the right hand with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. On this line are put several brass center pins or dots; the first is placed at 2150,42 in the first radius, and marked **M B**, signifying the cubic inches in a bushel of malt; the second is at 282 in the same radius, and marked **A G**, signifying the number of

of cubic inches in a gallon of ale; the like marks and dots are again repeated at the same numbers in the second radius.

2. The second line on this face is that on the slide, and is known by the letter **B**, which is also a double line of numbers, of the same length and divided in the same manner, as the line **A**; so that when unity at the end of one is applied to unity at the end of the other, all the figures of the one are parallel to those of the other, and each division and subdivision of the one are collateral to those of the other. On this line are also placed several brass pins or dots; the first is at 231, marked **wg** in the first radius, signifying the cubic inches in a gallon of wine; the second is at 3,14, and marked **c**, signifying the circumference of a circle whose diameter is unity, which marks are again repeated in the second radius; the third dot is placed at 707, and marked **si**, signifying the side of the greatest square inscribed in a circle whose diameter is unity; the fourth is at ,886, and marked **se**, signifying the side of a square equal in area to that of a circle whose diameter is unity.

3. The third line on this face is that on the rule called the malt line, and known by the letters **MD**, signifying malt depth, which is also a double line of numbers of the same length, and divided in the same manner as the two former, but is broken and inverted, beginning at 2150,42, and is numbered from the left to the right hand inversely, *viz.* 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, 6, 5, 4, 3, and ends at 2150,42, where it began.

Of the LINES on the second face or side of the RULE.

1. **T**HE first line on this side is put on the rule, and known by the letter **D**, which is a line of numbers of a single radius, and is numbered on from 1 at the beginning to 10 at the end. This line of numbers of a single radius is just the same length with the lines **A B**, and **MD**, before described. On this line also are placed certain

certain gage points; as at 17,15 is marked **W G**, signifying the circular gage point for wine gallons; at 18,95 is marked **A G**, signifying the circular gage point for ale gallons; at 46,37 is marked **M S**, signifying the square gage point for malt bushels; at 52,32 is marked **M R**, signifying the round or circular gage point for malt bushels; and at 6,23 is marked **tp**, signifying the circular gage point for a pound of net tallow.

Note, On this line may all the other gage points, mentioned in page 54, be found, although they are not pointed out by letters and dots as those before mentioned.

2. The second line on this face is that put upon the slide, and known by the letter **c**, which is a line of numbers of a double radius, equal in length to the line **D**, and also to the lines **A** and **B**, and it is divided exactly in the same manner as the lines **A** and **B**.

When the lines **D** and **c** are set even at both ends, *viz.* unity at the beginning of the one to unity at the beginning of the other, then the line **c** represents the squares, and the line **D** the roots of those squares.

3. The third and fourth lines on this face are two lines of segments for ullaging of casks, and are set on the rule; one of which is marked **S'**, that is, segments lying, and the other **S**, segments standing; and they are both alike numbered from the left hand to the right, with the figures 1, 2, 3, &c. to 10, and from 10, 20, 30, 40, to 100.

When the line **E** is put upon the rule, it is placed here in the room of the two lines of segments, and those segments are put on one of the edges of the rule with a line of numbers of double radius to slide between them; and in this case it will be proper to have the line **D** upon the slide, and the line **c** in **D**'s place on the rule, so as that the line **D** may slide between the lines **c** the squares, and **E** the cubes, for the line **E** is a triple line of numbers, consisting of three radiusses, and is equal in length to both the single and double lines of numbers: and as those numbers on **c** are squares to those on **D**, so are the numbers on the line **E** cubes to those

on **D**; so that if the line **D** on the slide be set even at both ends with the lines **C** and **E**, you have a table of cubes, squares, and their roots, *viz.* on **E** are the cubes, on **C** are the squares, and on **D** the roots.

Of the lines on the third face or edge of the rule.

1. **T**HE first line on this face or edge is a line of inches decimally divided, and numbered on from 1, 2, 3, to 12.

2. The second line on this edge is that marked spheroid, and is numbered on from 1, 2, 3, to 7, which is a line of differences for reducing the first variety of casks to a mean diameter.

3. The third line on this edge is that marked 2^d variety, and is numbered on from 1, 2, to 6; which is also a line of differences for reducing the second variety of casks to a mean diameter.

4. The fourth line on this edge is marked 3^d variety, and is likewise a line of differences for reducing the third variety of casks to a mean diameter.

Of the lines on the fourth face or edge of the rule.

1. **T**HE first line on this edge is marked **F M**, signifying foot measure, and is numbered on from 1, 2, 3, to 10, which is for reducing of inches, &c. to the decimal parts of a foot.

2. The second line on this edge is a line of inches decimally divided, and numbered on from 1, 2, 3, to 12.

3. The last line on this edge is that marked **F G**, signifying the frustums of two cones, and is numbered on from 1, 2, 3, to 6, which is a line of differences for reducing the fourth variety of casks to a mean diameter.

These are all the lines that are usually put on the outside faces of the rule; on the inside, or underneath the slides, there are generally put a line of inches, with their respective areas in ale gallons at those several diameters; but they being of so little use in practice, I shall take

take no more notice of them ; besides, I think their place might be supplied with something that would be of much more service.

There are now made a more convenient sort of sliding rules than that before described, which were first invented by Mr. Verie, then collector of excise. This rule consists of four slides with a single radius, and is so contrived that two of the slides work together ; by which means one of this sort made 6 inches long answers another made the common way of 12 inches, or one of 9 inches to another of 18 inches ; whereby the divisions are enlarged, and the rule rendered much more portable.

On each of the four slides is put the line of numbers, commonly called Gunter's line, with all the usual marks and dots as on the common rules ; two of which are for sliding together between the line D, which is broken into two parts, and the lines A and M D ; the other two are for sliding between the lines of segments, which are also broken into two parts. And although the line D and the two lines of segments are broken into two parts, yet they are so fitly placed on the sides of the rule, that the two slides may slide together between them in such a manner as that any question may be as readily solved as if they were in one continued line.

Underneath the slides are put the lines of varieties, the divisors and gage points ; and also the several factors for reducing of goods of one denomination to its equivalent value in those of another.

There is also a table of the malt divisor 2150, which is doubled, trebled, &c. to 9 times, for estimating the content of a floor of malt. If the rule before described be thoroughly understood, this will be known at the first sight.

Thus have I described and explained all the lines that are usually put on the sliding rule, and in the next place intend to shew their uses.

But here I would observe that all those lines are put on the *Branan's* rule, except the lines of varieties, whose place may be supplied with proper factors that would

answer the same end, and may be used in every respect as a sliding rule, and even with more advantage in some cases; for by means of a new contrivance the cube line or line E on the *Branan's* rule may be made so as to slide by the line D, or the roots, which line is not very common to be met with on the sliding rule.

A R T. II.

Of Numeration by the line of numbers.

1. **T**HE line of numbers is divided into nine unequal parts, called primes, which are distinguished by the nine digits, beginning at unity and ending at unity; and each of those divisions are again divided into ten other unequal parts, called tenths; and each of those tenths are subdivided or supposed to be divided into ten other parts, called centesms, according as the length of the line or radius will bear.

Every tenth from 1 to 2 on the line D is actually divided into centesms; but from 2 to 3 the tenths are only divided into five parts, so there each division contains two centesms. From 3 to the end of the line D each tenth is divided but into two parts, and each of these parts contains five centesms.

2. In the double lines of numbers, *viz.* the lines A, B, and C, the tenths from 1 to 2 will admit of only five divisions, each of which divisions contains two centesms. From 2 to 5 each tenth is divided into but two parts, each part containing five centesms; and from 5 to 10 the tenths will not admit of any division at all, without crowding one another, therefore the centesms must be guessed at in those places.

3. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are all arbitrary points, and may be conceived to represent so many entire units, tens, hundreds, or thousands.

Or they may represent so many tenth, hundredth, or thousandth parts of an unit.

If the figure 1 at the beginning of the line represent
one

one unit, then will the figures 2, 3, 4, 5, 6, 7, 8, 9, represent so many units, and the tenths in each of those primes will be the tenth part of unity; the centesms in each of those tenths will be the hundredth parts of unity.

Or if 1 at the beginning of the line represent 10 units, then will each figure forward towards the right hand represent ten times as many units as the figure expresses. As for example, if the figure 1 at the beginning of the line represents 10 units, then will the figure 2 represent 20 units, the figure 3 30 units, and the figure 4 40 units, the figure 5 will represent 50 units, &c. till you come to 1 or 10 at the end of the radius, which will represent 100 units.

Or the figure 1 at the beginning of the line may represent one tenth part of an unit, then each prime towards the right hand will represent so many tenth parts of unity, and each tenth in these primes will be hundredth parts of unity, and each centesm in those tenths will be the thousandth parts of unity.

That this may be the better conceived, I shall subjoin the following schemes :

1. For the single radius, or line D.

From 1 to 2 the tenths are divided into ten parts, called centesms : therefore

If the figure 1 at the beginning of the line represent	.1	the larger divisions will represent	.01	& the smaller divisions will represent	.001
	1.		.1		.01
	10.		1.		.1
	100.		10.		1.

From 2 to 5 the tenths are divided into five parts, each part containing two centesms : therefore

If the figure 2 represent	.2	the larger divisions will represent	.01	& the smaller divisions will represent	.002
	2.		.1		.02
	20.		1.		.2
	200.		10.		2.

From 3 to the end of the line D, the tenths are only divided into two parts, each containing five centesms : therefore

Of the Sliding Rule.

If the figure 3 represent $\left\{ \begin{array}{l} .3 \\ 3. \\ 30. \\ 300. \end{array} \right\}$ the larger divisions will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$ & the smaller divisions will represent $\left\{ \begin{array}{l} .005 \\ .05 \\ .5 \\ 5. \end{array} \right\}$

2dly. For the lines A, B, and C, of double radius.

From 1 to 2 the tenths are divided into five parts, each part containing two centesms; therefore

If the figure 1 at the beginning of the line represent $\left\{ \begin{array}{l} .1 \\ 1. \\ 10. \\ 100. \end{array} \right\}$ the larger divisions will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$ & the smaller divisions will represent $\left\{ \begin{array}{l} .002 \\ .02 \\ .2 \\ 2. \end{array} \right\}$

From 2 to 5 the tenths are divided into two parts, each part containing five centesms; therefore

If the figure 2 represent $\left\{ \begin{array}{l} .2 \\ 2. \\ 20. \\ 200. \end{array} \right\}$ the larger divisions will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$ & the smaller divisions will represent $\left\{ \begin{array}{l} .005 \\ .05 \\ .5 \\ 5. \end{array} \right\}$

From 5 to 10 the primes are only divided into ten parts, so the centesms must be guessed at :

Therefore, if the figure 5 represent $\left\{ \begin{array}{l} .5 \\ 5. \\ 50. \\ 500. \end{array} \right\}$ the larger divisions, or tenths, will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$

3dly. For the line E of triple radius.

From 1 to 4 the tenths are divided into two parts, each part containing five centesms; therefore

If 1 at the beginning of the line represent $\left\{ \begin{array}{l} .1 \\ 1. \\ 10. \\ 100. \end{array} \right\}$ the larger divisions will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$ & the smaller divisions will represent $\left\{ \begin{array}{l} .005 \\ .05 \\ .5 \\ 5. \end{array} \right\}$

From 4 to the end of the radius, the tenths; for want of room, are not divided at all :

Therefore, if the figure 4 represent $\left\{ \begin{array}{l} .4 \\ 4. \\ 40. \\ 400. \end{array} \right\}$ then the larger divisions, or tenths will represent $\left\{ \begin{array}{l} .01 \\ .1 \\ 1. \\ 10. \end{array} \right\}$

By

By the foregoing schemes it will be an easy matter to conceive what the tenths and centesms represent, when the value of the primes are fixed.

Here it will not be amiss to observe, 1. Of whatsoever denomination 1 at the beginning of the single radius or line D is of, the 10 at the end thereof will be ten times as many.

2. Of whatsoever denomination the 1 at the beginning of the lines of double radius, *viz.* the lines A, B, and C are of, the 1 in the middle is ten times as many, and the 10 at the end thereof is a hundred times as many.

3. Of whatsoever denomination the 1 at the beginning of the triple line, or line E is of, that the next 1 towards the right hand is ten times as many, and the third 1 or 10 is a hundred times as many, and the last 1 or 100 is a thousand times as many.

By what has been said I imagine that it will be no difficult matter to find the point upon any of the lines where any given number is represented. As for example, suppose 17,15 were required to be found upon the line D.

For the first figure (*viz.* 1) count 1 at the beginning of the line D for 10, for the second figure 7, count 7 of the following larger divisions, or tenths, for seven units, which is the point where 17 stands; then from this point forward count one centesm from the third figure 1, and for the last figure 5 count half the next centesm; and at that point you will find the dot marked with W G, which represents 17,15, which was required.

If it had been required to find ,1715 1,715 171,5 or 1715, they would all be found at the same point: but then the figure 1 at the beginning of the line would be conceived of a different value in finding each of the above numbers, *viz.* .1 1. 10. 100, 1000.

By the same rule will the numbers 18,95 be found at the point A G. 46.37 at M S. 52.32 at M. R. and 6.23 at tp.

And in this manner is the point that represents any
C 4 other

other number found either upon this line or any of the others.

But it will be proper to observe the following directions:

1. If the given number consist of three figures, the middle being a cypher, then you must find it in the first tenth following the prime of that number. For example, let the given number be 406, the first figure 4 = 400 find at the fourth prime; the second figure being a cypher, count no tenths; but for the last figure 6 count six centesms, which is a little to the right hand of the lesser division in that tenth, and that is the point where 406 is represented. If the number 404 had been to be found, it would have been found the same distance from the lesser division, but on the other side towards the prime.

2. If the given number consists of four places of figures, having two cyphers in the middle, then it will be found near the prime unto which it belongs, *viz.* between the prime and the next centesm following towards the right hand. As for example, If the number 2005 were to be found, the first figure 2 = 2000 is found at the beginning of the second prime, and because there are cyphers in the second and third places, you must not count any of the tenths or centesms, but for the last figure 5 count half of the first centesm towards the right hand, and that will be the point where 2005 is represented.

3. If the given number consist of 2, 3, or 4 places of figures, and all of them cyphers, except the first, then they are all found at the beginning of the prime unto which they belong; so that if the numbers were ,003, ,03, 30, 300, 3000, they are all found at the same point, *viz.* at the beginning of the third prime; or if the numbers were ,004, ,04, 40, 400, or 4000, they are all found at the same point, *viz.* at the beginning of the fourth prime; and so of any other number. Therefore if what has been already said concerning numeration on the rule be duly considered, and thoroughly understood, it will

will be sufficient for the finding the point on the rule where any number whatsoever is represented, without adding any more examples; so I shall proceed in the next place to shew the application of those lines to multiplication.

ART. III.

Of Multiplication by the lines A and B.

WHEN two numbers are given to be multiplied together, whether they be whole numbers or decimal fractions. The proportion is,

As unity on B is to the multiplicator on A, so is the multiplicand on B to the product on A.

Or,

As unity on A is to the multiplicator on B, so is the multiplicand on A to the product on B.

EXAMPLE I.

Let the product of 8 by 6 be sought.

OPERATION.

Set 1 on B to 6 on A, and against 8 on B is 48 upon A, the product required.

Or,

As 1 on A is to 6 on B, so is 8 on A to 48 on B, the same as before.

Thus you see the first term or unity may be taken upon either of the lines A or B, provided the third term be taken upon the same line as the first, and the fourth term on the same line as the second.

EXAMPLE II.

What is the product of 65 by 18.

OPERATION.

Set 1 on B to 18 on A, and against 65 on B is 1170 on A, the product required.

C 5

EXAM-

EXAMPLE III.

What is the product of 76 by 13?

OPERATION.

Set 1 on B to 13 on A, and against 76 on B is 988 on A, the product required.

And in this manner may the product of any other two numbers be found; but in order to know how many figures the product will consist of, observe the following directions.

1. If the first figures of the product are less than the least of the two factors, the product will have as many places as both factors.

2. If the first figures of the product are equal, or exceed the least of the factors, then the product will have one place less than the two factors.

These rules are manifest by the foregoing examples; for in the first example the least of the factors is 6, and the first figure of the product is 4, which is less than 6; therefore the product is 48, that is, it consists of two places of figures equal to the number of places of both factors. And in the second example likewise the least of the factors is 18, and the two first figures of the product are 11, which being less than 18, therefore the product consists of four places of figures, and is = 1170, equal to the number of places of both factors. But in the third example the least of the factors is 13, and the two first figures of the product are 98, which is greater than 13; therefore the product consists of but three places of figures, and is = 988, that is, one place less than both factors.

The next thing to be considered is, to know the value of the products, whether they be whole numbers, mixt numbers, or decimal fractions; as for instance, in the last example, where the product was found to be 988, and not 98,8, or ,988, as is plain by the following table; for set 1 on B to 13 on A,

Then

$$\text{Then against } \left\{ \begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 7.6 \end{array} \right\} \text{ upon B is } \left\{ \begin{array}{l} 26 \\ 39 \\ 52 \\ 65 \\ 78 \\ 91 \\ 98.8 \end{array} \right\} \text{ upon A.}$$

And if the numbers on B be multiplied by 10, those also upon A will be multiplied by 10, that is, if 2 be supposed to be 20, then will 26 be equal to 260; and if 3 be supposed to be = 30, then will 39 = 390, and so of all the rest; and 7.6 will become 76, and 98.8 will be = 988, which is the true product, as before found. If 76 had been a decimal, *viz.* .76, then the product would have been 9.88; or if it had been ,076, then the product would have been .988.

When one of the factors is a whole or mixed number, and the other a decimal fraction, then set unity on the middle of B to the whole or mixt number on A, and seek the fraction toward the left hand on B, because it is less than unity, and against it is the product on A, which is always less than the given whole or mixt number. For example,

What is the product of 45.3 by .7?

OPERATION.

Set 1 on the middle of ~~A~~ to 45.3 on A, and against 7 on B is 31.71 on A, the product required. I might here add more examples, but I think the foregoing examples and directions are sufficient for any variety that can occur, therefore I shall proceed to the next article of division by lines.

ART. IV.

Of Division by the lines A and B.

WHEN one number is given to be divided by another, whether whole numbers, mixt numbers, or decimal fractions, the proportion is,

C 6

As

As the divisor upon A is to unity upon B, so is the dividend on A to the quotient on B.

Or,

As the divisor on B is to unity on A, so is the dividend on B to the quotient on A.

EXAMPLE I.

What is the quotient of 48 divided by 6?

OPERATION.

Set 6 upon A to unity on B, then against 48 on A is 8 upon B, the quotient required.

Or,

Set 6 upon B to unity on A, then against 48 on B is 8 on A, the same as before.

EXAMPLE II.

What is the quotient of 1170 by 18?

OPERATION.

Set 18 upon A to unity on B, then against 1170 on A is 65 upon B, the quotient required.

EXAMPLE III.

What is the quotient of 988 divided by 13.

OPERATION.

Set 13 on A to unity on B, then against 988 on A will be 76 on B, the quotient required.

Any other quotient may be found in like manner; but in order to discover how many figures the quotient will consist of, observe the following rule:

The quotient must consist of as many figures as there is difference of figures between the dividend and the divisor, except when the first figures of the dividend are equal to, or more than the divisor, and then it must have one place more.

This will appear from the foregoing examples; for in the first and second examples the divisors were greater than the first figures of the dividends, *viz.* 8 is greater than 6, and 18 is more than 11, the two first figures of the dividend in the second example,

There-

Therefore the quotients in both examples are equal to the difference of the number of figures in the divisor and the dividend; but in the third example the two first figures of the dividend = 98, are greater than the divisor = 13; therefore the quotient consists of two figures, that is, of one place more than the difference between the divisor and the dividend. And thus it may be determined, in all cases, of how many places of figures the quotient shall consist.

And the value of the quotient will appear in this manner: for instance, when 13 on A is set to unity on B, as in the last example.

$$\text{Then against } \left\{ \begin{array}{l} 26 \\ 39 \\ 52 \\ 65 \\ 78 \\ 91 \\ 98.8 \end{array} \right\} \text{ upon A is } \left\{ \begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 7.6 \end{array} \right\} \text{ upon B.}$$

And if those numbers upon A are supposed to be multiplied by 10, those also on B will be multiplied by the same; for if we count 26 = 260, then will 2 = 20; if 39 be called 390, then will 3 = 30; if 52 be reckoned 520, then will 4 = 40, &c. till you come to 98.8, which multiplied by 10 = 988, and then 7.6 will become 76 = the true quotient before found. From hence it is manifest that at once setting the rule, both multiplication and division are performed; for the last example is the converse of the third example in multiplication, and when the rule is so set, the lines A and B are in effect tables of products and quotients; for if 13 be taken as a multiplicator, set unity on B to 13 on A, then against any multiplicand on B you have the product upon A, as against 2 is 26, against 3 is 39, against 4 is 52, &c. Again, if you suppose 13 a divisor, as the rule now stands against any dividend on A, is the quotient on B, as against 26 on A is 2 on B, against 39 on A is 3 on B, against 52 on A is 4 on B, &c. By which it is evident that

that multiplication and division are but the converse of one another, so that if the one is understood, the other cannot be unknown.

The foregoing rules and examples I imagine will be sufficient to give the young gager an insight into this matter, therefore I shall add no more on this head, but proceed to the next.

ART. V.

Of the Single Rule of Three direct by the lines A and B.

IN the single rule of three direct, there are always three numbers given to find a fourth in direct proportion.

The proportion is,

As the first term upon A is to the second term on B, so is the third term upon A to the fourth term on B.

Or,

Set the first term upon B to the second on A, and against the third term on B is the fourth term upon A.

Note, the first and third terms must always be taken on the same line, *viz.* on either A or B, and the second and fourth terms must be found on the other.

EXAMPLE I.

If 8 yards of cloth cost 48s. what will 18 yards cost at that rate?

OPERATION.

on A.	on B.	on A.	on B.	
As 8	: 48	: :	18	: 108 = Answer.
Yds.	Shills.		Yds.	Shills.

Or,

on B.	on A.	on B.	on A.	
As 8	: 48	: :	18	: 108 = the same as before.

By which it appears, that it matters not on which line you take the first term, provided you take the third term on the same line, and the second and fourth term on the other line.

E X A M -

EXAMPLE II.

If 3 gallons of ale cost 2s. what will 36 gallons cost at that rate?

OPERATION.

on B.	on A.	on B.	on A.	
As 3	: 2	: 36	: 24	= Answer.
Galls.	Shills.	Galls.	Shills.	

EXAMPLE III.

What is the weight of 30 rods of candles of 12 to the pound, when there are 24 candles on each rod?

OPERATION.

on B.	on A.	on B.	on A.	
As 12	: 30	: 24	: 60	= Answer
Numb. of Rods.		Numb. of Weight;		
Candles.		Candles.		

More examples might be adduced, but these being well understood, will be sufficient to work any proportion whatsoever.

A R T. VI.

Of Inverse Proportion, or the Single Rule of Three indirect by the lines A and B.

INverse proportion is when there are three numbers given to find a fourth number that shall bear the same proportion to the second term as the first term does to the third, which is known by the nature of the question, as when more requires less, or less requires more.

EXAMPLE I.

If 8 men can do a piece of work in 20 days, how long will 4 men take to perform the same?

PROPORTION.

	1	2	3	4	Terms.
As 8		20	: 4	: 40	

Operation

Of the Sliding Rule.

Operation by the Sliding Rule.

on B. on A. on B. on A.
 As 4 : 8 : : 20 : 40 = Answer.
 Men. Men. Days. Days.

By the nature of this question, it is evident that it is inverse proportion; for less men must require more time to perform the same piece of work.

EXAMPLE II.

If 4 men can do a piece of work in 40 days, how many men will do the same work in 20 days?

PROPORTION.

$\begin{matrix} 1 & 2 & 3 & 4 \text{ Terms.} \\ \text{As } 40 & : & 4 & : : 20 & : & 8 = \text{Answer} \end{matrix}$

By the Rule.

on B. on A. on B. on A.
 As 20 : 40 : : 4 : 8 = the same as above.

Here it is also evident, that less time requires more men to perform the same work in. And in this manner are such kind of questions discovered to be in the inverse rule of proportion.

ART. VII.

To reduce a vulgar fraction to its equivalent decimal expression.

The proportion is

AS the denominator upon A is to unity on B, so is the numerator on A to the decimal required on B.

EXAMPLE I.

Reduce $\frac{1}{4}$ to a decimal fraction.

OPERATION.

on A. on B. on A. on B.
 As 4 : 1 : : 1 : .25 = Decimal sought.
 Denom. Unity. Num. Decimal.

EXAM-

EXAMPLE II.

Reduce $\frac{9}{11}$ to a decimal fraction.

OPERATION.

on A,	on B.	on A.	on B.	
As 12	: 1	: :	9	: ,75 = Decimal sought.
Denom.	Unity.	Num.	Decimal.	

EXAMPLE III.

Reduce $\frac{13}{18}$ to a decimal fraction.

OPERATION.

on A.	on B.	on A.	on B.	
As 28	: 1	: :	18	: ,643 fere = Dec. sought.
Denom.	Unity.	Num.	Decimal.	

And in like manner may any other vulgar fraction whatsoever be reduced to its equivalent decimal expression.

ART. VIII.

To find a factor to a divisor that shall perform the same by Multiplication as the divisor would do by Division.

The Proportion is

AS the divisor given is to unity, so is unity to the factor required.

EXAMPLE I.

Suppose 25 to be the divisor, what will be the factor to that number.

OPERATION.

on A.	on B.	on A.	on B.	
As 25	: 1	: :	1	: ,04 = Factor sought.
Divisor.	Unity.	Unity.	Factor.	

PROOF.

Let 525 be divided by 25, the divisor, multiplied by ,04, the factor above found.

By

By Division.

$$\begin{array}{r} 25 \overline{) 525} \quad (21 = \text{Quotient.} \\ \underline{50} \\ 25 \\ \underline{25} \end{array}$$

By Multiplication.

$$\begin{array}{r} 525 \\ \times 04 \\ \hline 21,00 = \text{Product.} \end{array}$$

By this you see that the same thing may be effected by Multiplication as by Division, for the quotient and the product are both equal.

EXAMPLE II.

What will the factor to 80 be?

OPERATION.

on A. on B. on A. on B.
As 80 : 1 :: 1 : ,0125 = Factor sought.
Div. Unity. Unity. Factor.

This being so easy, it would be needless to add any more examples.

ART. IX.

Having a factor given to find a divisor.

THIS is the converse to the last article, therefore the proportion is,

As the factor is to unity, so is unity to the divisor required.

EXAMPLE I.

Let ,04 be the factor given to find a divisor.

OPERATION.

on A. on B. on A. on B.
As ,04 : 1 :: 1 : 25 = Divisor sought.
Factor. Unity. Unity. Divisor.

EXAMPLE II.

Let ,0125 be the given factor, what will be the divisor?

O P E.

OPERATION.

on A. on B. on A. on B.

As, 0125 : 1 : : 1 : 80 = Divisor sought.

Factor. Unity. Unity. Divisor

And in like manner may any other divisor be found to any given factor.

ART. X.

To extract the square root by the lines c and d.

THIS is performed by inspection when the lines **c** and **d** are applied even at both ends, as I observed in page 24; for when unity at the beginning of **c** is set even to unity at the beginning of **d**, you have a table of squares and their roots; for against any square number upon **c** you have the root upon **d**, and on the contrary against any root on **d** you have the square on **c**; but here it will be necessary to observe the following directions:

1. If the given square number consists of an odd number of places of integers, seek it on the first radius of the line **c**, and against it you have the root on **d**.

2. If the given square number consists of an even number of places of integers, then seek it in the second radius of the line **c**, and against it on the line **d** is the root required.

Examples in the 1st case.

Against $\left\{ \begin{array}{l} 9 \\ 144 \\ 20736 \\ 90000 \end{array} \right\}$ in the 1st radius upon the line **c** is $\left\{ \begin{array}{l} 3 \\ 12 \\ 144 \\ 300 \end{array} \right\}$ the root on the line **d**.

Examples in the 2d case.

Against $\left\{ \begin{array}{l} 36 \\ 19.46 \\ 2150.42 \\ 250000 \end{array} \right\}$ in the 2d radius of the line **c** is $\left\{ \begin{array}{l} 6 \\ 4.4 \\ 46.37 \\ 500 \end{array} \right\}$ the root on the line **d**.

ART.

ART. XI.

To extract the cube root by the lines D and E.

THIS is also done by inspection, when the lines *D* and *E* are set even at both ends, *viz.* unity at the beginning of *D* to unity at the beginning of *E*. In this position those lines are in effect a table of cubes and their roots, for against any cube on *E* is the root upon *D*; and on the contrary, against any root on *D*, is the cube upon *E*. The only difficulty in this work is to know on which of the three radiusses of the line *E* you are to find your cube number, and in which there are three cases.

CASE I.

If the given number consists of 1, 4, or 7 places of integers, find it on the first radius of the line *E*, and against it is the root on *D*.

CASE II.

If the given number consists of 2, 5, or 8 places of integers, find it on the second radius of the line *E*, and against it on the line *D* is the root required.

CASE III.

When the given number consists of 3, 6, or 9 places of integers, then find it on the third radius of the line *E*, and against it on *D* is the root required.

Examples in the 1st case.

	Cubes.		Roots.
Against	{ 3.375 }	on the 1st ra-	{ 1.5 }
	{ 8. }	dus of the	{ 2. }
	{ 1728. }	line <i>E</i> is	{ 12. }
			on <i>D</i> .

Examples in the 2d case.

	Cubes.		Roots.
Against	{ 27 }	on the 2d ra-	{ 3. }
	{ 35.937 }	dus of the	{ 3.3 }
	{ 10648. }	line <i>E</i> is	{ 22. }
			on <i>D</i> .

Exam-

Of the Sliding Rule.

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Examples in the 3d case.

	Cubes.	Roots.	
Against	$\left\{ \begin{array}{l} 166.375 \\ 729 \\ 274925 \end{array} \right\}$	$\left\{ \begin{array}{l} 5.5 \\ 9 \\ 65 \end{array} \right\}$	on the 3d ra- dius of the line E is on D.

Thus I have given rules and examples in all the different cases for finding of the cube root, which, if understood, will be sufficient to find the root of any number whatsoever.

I suppose I need not acquaint my reader that the number of integers in the root are discovered by pricking off every third figure (beginning at the unit's place) towards the left hand; and as many points as there are in the cube, so many places of integers there will be in the root; because I presume that he has learned to extract the cube root by the pen before he attempts to do it by the rule.

ART. XII.

Of the use of the line E on the sliding rule, in working of questions in triplicate proportion,

HERE I will make use of the same examples as those which were wrought by the pen in pages 20 and 21, shewing the use of the cube root; by which will appear the excellency of those lines for working such kind of questions.

QUESTION I.

If an iron bullet of four inches diameter weigh 9 pounds what will an iron bullet of 8 inches diameter weigh?

OPERATION.

on D.	on E.	on D.	on E.	
As 4	: 9	: 8	: 72	= Answer.
Diam.	Wt.	Diam.	Wt.	

QUES-

QUESTION, II.

If a fathom of rope of 10 inches circumference weigh 17 pound, how much will a fathom weigh of 8 inches circumference?

OPERATION.

on D.	on E.	on D.	on E.	
As 10	: 17	:: 8	: 8.704	= Answer.
Diam.	Wt.	Diam.	Wt.	

QUESTION III.

Given the dimensions of a conical tun, and the content of the same, *viz.* the top diameter = 30 inches, bottom diameter = 40 inches, depth = 20 inches, and the content 68,7 ale gallons. To find the dimensions of another tun of like form, that would contain 90 gallons.

OPERATION.

1. For the top diameter.

on E.	on D.	on E.	on D.	
As 68,7	: 30	:: 90	: 32,8	= Top diameter sought.
Cont.	Diam.	Cont.	Diam.	

2. For the bottom diameter.

on E.	on D.	on E.	on D.	
As 68,7	: 40	:: 90	: 43,8	= Bottom diameter sought.
Cont.	Diam.	Cont.	Diam.	

3. For the depth.

on E.	on D.	on E.	on D.	
As 68,7	: 20	:: 90	: 21,9	= Depth required.
Cont.	Depth.	Cont.	Depth.	

And thus are the answers found to be the same as those found by the pen, with a vast deal more ease and expedition.

CHAP. V.

Of a CIRCLE.

Definitions of a circle, and some of its properties, with the relation they bear to each other

1. **A** Circle is the most capacious of all figures that is, it comprehends the most space in the shortest boundary line of any figure whatsoever.

2. The areas of all circles are in proportion to one another as the squares of their diameters.

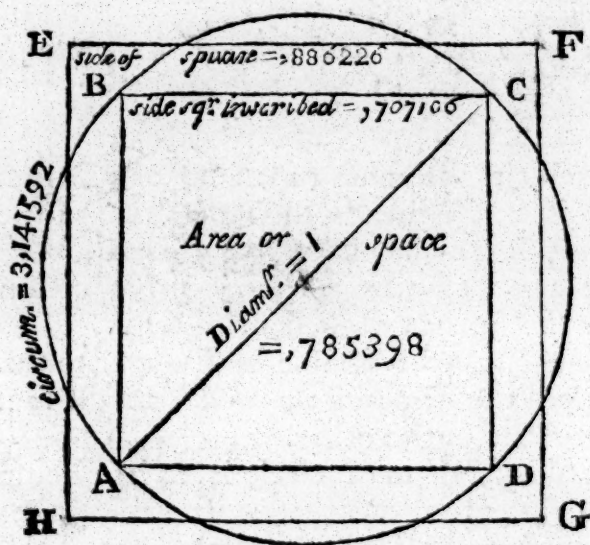
3. If one circle be double in diameter to that of another circle, the same circle will contain four times the area or space of the other.

4. The diameter of a circle is to its circumference as 1 is to 3.141592, &c. that is, supposing the diameter of a circle to be 1, the circumference of that circle will be 3.141592, &c. This proportion of the diameter to the circumference of a circle, was first found by one *Van Ceulen*, a *Dutchman*, who, with a great deal of labour and trouble, found it true to 36 places of decimals. Others, since him, have found it true to 100 places of decimals; but the first 6 decimals being sufficient for most purposes, therefore I have omitted the rest. And here it must be observed, that upon this supposition are grounded all the proportions relating to a circle in the following work, *viz.* If the diameter of a circle be 1, the circumference of that circle will be 3.141592.

Having premised these things, I shall, in the next place, shew the method of finding the relation that the side of the greatest inscribed square, the area, and the
side

side of the square equal, bears to the diameter of a circle.

Let the diameter of the circle A, B, C, D , annexed $= 1 = AC$, and the circumference $= 3,141592$, to find the side of the square inscribed $= AB = BC = CD = DA$, the area or space comprehended in the circle, and the side $EF = FG = GH = HE$ of the square equal.



1. To find the side of the greatest inscribed square $= BC$.

RULE.

It is demonstrated in 1 *Euclid* 47, that in every right angled triangle the squares of the hypotenuse is equal to the sum of the squares of the other two sides of the triangle. Now in the triangle, ABC , AB and BC are the two sides of the triangle, as well as the two sides of the inscribed square, and the diameter $AC = 1$ is the hypotenuse; therefore half $1 = ,5$, whose square root is $= BC$, or AB required.

OPE-

OPERATION.

,50),707106 = Side of the inscribed square.

49

$$\begin{array}{r} 1407 \overline{) 10000} \\ \underline{9849} \end{array}$$

$$\begin{array}{r} 14141 \overline{) 15100} \\ \underline{14141} \end{array}$$

$$\begin{array}{r} 1414206 \overline{) .9590000} \\ \underline{8485236} \end{array}$$

Rem. 1104764

2. To find the area of a circle whose diameter is unity, or the area of any other circle.

RULE.

Multiply half the circumference by half the diameter and the product will be the area required.

OPERATION.

Half of 3.141592 = 1.570796

Half of 1 = ,5

Area of a circle = ,7853980 whose diameter = 1.

3. To find the side of a square equal in area to that of a circle whose diameter is unity, equal to EF = FG, &c.

RULE.

If the side of a square be multiplied by itself, the product will be the area of that square; and having already found that, when the diameter of a circle is 1, the area is = ,785398; therefore extract the square root of that area, and it will be the side of the square equal required = EF = FG, &c.

D

OPE-

OPERATION.

2 $\dot{7}85398) .886226 = \text{Side of a square equal.}$
64

168) 1453
1344

1766) 10998
10596

17722) .. 40200
35444

177242) 475600
354484

1772446) 12111600
10634676

Remains .1476924

One of the sides of a square equal in area to that of a circle whose diameter is 1, being ,886226, which $\times^a$ by 4 = 3.544904, the sum of the four sides. Now the circumference of the same circle is but 3.141592, which is less by ,403312 than the sides of a square equal; which proves that the circle is a more capacious figure.

Having found that if the diameter of a circle be 1,

{	The circumference will be	—	3.141592
	The side of the greatest inscribed square		,707106
	The side of a square equal	— —	,886226
	The area	— — — —	,785398

I shall now proceed to shew the uses of these factors in finding, the like parts of a circle of any other diameter, both by pen and rule.

Suppose the diameter of a circle was 30 inches, what will the circumference, the side of the square inscribed,

inscribed, side of a square equal, and area of that circle be?

1. For the circumference by the pen.

RULE.

Multiply the circumference of the circle whose diameter is unity by the given diameter, and the product will be the circumference of the circle required.

OPERATION.

Circumference of a circle whose diameter } 3.141592
 = 1 = — — — — — }
 Given diameter — — — — — 30

Circumference required = ————— 94.247760

By the Rule.

Set unity on A to 3,14 on B, and against 30 on A is 94,24 on B, the same as before.

The rule being thus set it is a table; for against any diameter on A is the circumference on B; *e contra*, against any circumference on B is the diameter upon A: as for example,

Against these diameters upon A	{	20	Are these circumferences upon B	{	62.83
		24			75.39
		27			84.82
		33			103.67
		36			113.09
		40			125.66

2. To find the side of the inscribed square.

Rule by pen.

Multiply the side of the inscribed square whose diameter is unity, by the given diameter, and the product will be the side of the inscribed square required.

OPERATION.

Side of the square whose diameter is 1 = .707106
 Given diameter — — — = 30

Side of the inscribed square required = 21.213100
 D 2 By

Of a Circle.

By the rule.

Set unity on A to ,707 on B, and against 30 on A is 21,21 on B, the same as before.

The rule, being thus set, is likewise a table; for against any diameter on A is the side of the square inscribed upon B; *e contra*, against any side of an inscribed square upon B is the diameter on A; as for example:

Against these dia-	$\left\{ \begin{array}{c} 20 \\ 24 \\ 27 \\ 33 \\ 36 \\ 40 \end{array} \right\}$	Are these sides	$\left\{ \begin{array}{c} 14.14 \\ 16.97 \\ 19.09 \\ 23.33 \\ 25.45 \\ 28.28 \end{array} \right\}$
meters upon A		upon B	

3. To find the side of a square equal.

Rule by pen.

Multiply the side of a square equal, whose diameter is unity, by the given diameter, and the product will be the side of the square equal required.

OPERATION.

Side of a square equal whose diameter is 1	=	,886226
Given diameter	—	— = 30
Side of the square equal required	—	= 26,586780

By the rule.

Set unity on A to ,886 on B, and against 30 on A is 26,58 on B, the same as before.

The rule being thus set, it is also a table, for against any diameter on A is the side of the square equal on B; *e contra*, against any side of a square equal on B is the diameter on A; as for example.

Against these dia-	$\left\{ \begin{array}{c} 20 \\ 24 \\ 27 \\ 33 \\ 36 \\ 40 \end{array} \right\}$	Are these sides of	$\left\{ \begin{array}{c} 17.72 \\ 21.26 \\ 23.92 \\ 29.24 \\ 31.90 \\ 35.44 \end{array} \right\}$
meters on A		a square = on B	
			4. To

4. To find the area.

Rule by the pen.

Multiply the area of a circle whose diameter is unity by the square of the given diameter, and the product will be the area required.

OPERATION.

$$\begin{array}{r} \text{Given diameter} = 30 \\ \quad \quad \quad 30 \\ \hline \end{array}$$

$$\text{Square of diameter} = 900$$

$$\text{Factor for the area} = ,785398$$

$$\text{Square of diameter} = \quad \quad \quad 900$$

$$\text{Area required} \text{ — } = 706,858200$$

By the rule.

Set unity on D to ,7854 on c, and against 30 on D is 706,8 on c, the same as before.

The rule being thus set, it is a table; for against any diameter on D is the area on c; and *e contra*, against any area on c is the diameter on D; as for example:

Against these diameters on D	$\left\{ \begin{array}{l} 2 \\ 4 \\ 8 \\ 16 \\ 32 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{Are those} \\ \text{areas on} \\ \text{c} \end{array} \right\}$	3.14
			12.56
			50.26
			201.06
			804.26

In the above examples the diameters are doubled every time, and the areas answering each double diameter are fourfold the other; which is a proof that any circle double in diameter to another, will comprehend four times the area or space of that other.

Having found a factor for the square of the diameter of any circle, *viz.* ,785398, there may be a divisor found by Chap. IV. Art. 9, that will answer the same end, and which, in many cases, will be more commodious; for if unity be divided by ,785398, the above

D 3

factor,

factor, the quotient will be 1.27324 the respective divisor, and it will be the same thing whether the square of the diameter is multiplied by ,785398, or divided by 1.27324; for both the product and quotient will be equal to the area in square inches, or in the same measure that the diameter was taken in. But on account of the different capacities of gallons, bushels, and feet, it will be necessary to find factors and divisors for each of these different denominations, because the dimensions are all taken in inches.



CHAP. VI.

Of finding divisors and gage points.

ART. I.

To find divisors and gage points for circles

ACCORDING to act of parliament there is contained in a *Winchester* gallon of ale 282 solid inches, in a gallon of wine 231 solid inches, and in a bushel of malt 2150.42 solid inches; consequently in a gallon of malt are contained 268.8 solid inches, which is the $\frac{5}{8}$ part of 2150.42; and it being found, that if the diameter of a circle be 1 inch, the area thereof will be ,785398, which may be called ,7854 decimal parts of an inch; therefore if the capacities of each gallon and bushel, &c. be divided by ,7854, the several quotients will be proper divisors for the square of any diameter to reduce the area into ale, wine, and malt gallons or bushels, &c. and the square roots of those divisors will be the gage points for circles.

See

See the Work.

Div ^r .	Div ^d .	Quot ^r .	S. R ^{ts} .	
,7854	282.0000	359.05	18.95	Ale gallons.
,7854	231.0000	294.12	17.15	Wine gallons.
,7854	268.8000	342.24	18.5	Malt gallons.
,7854	2150.4200	2738.	52.32	Malt bushels.
,7854	227.0000	289.	17	Mash tun gallons.

The quotients are the divisors, and the square roots the gage points. In like manner may any other divisor or gage point be found, when the solid capacity of the integer is given, whether it be a gallon, a bushel, a pound weight, or a foot, &c. And in this manner were all the divisors and gage points in the following table found.

ART. II.

To find factors for circles.

SINCE the same may be performed by Multiplication as by Division, and there may be factors found which will answer the same end as the divisors before found; therefore, to find the same, divide ,7854 by the solid capacities of each gallon and bushel, &c. and the quotients will be proper multipliers for the square of the diameter of any circle to reduce the area of that circle into ale, wine, malt gallons, or bushels, &c.

See the work.

Div ^r .	Div ^d .	Quot ^r .	
282.	,7854	,002785	Common factors for
231.	,7854	,003399	the square
268.8	,7854	,002922	of the dia-
2150.42	,7854	,000365	eters for
227.	,7854	,00346	

{ Ale gallons.
 { Wine gallons.
 { Malt gallons.
 { Malt bushels.
 { Mash tun gall.

In like manner were the other factors for circles found, which are inserted in the following table.

ART. III.

To find factors for squares.

THIS is done by dividing of unity by the solid capacity of each gallon, bushel, foot, pound, &c. See the following examples.

282.	1,000000	{,003546}	Common factors for squares.	{	Ale gallons.
231.	1,000000	{,004329}			Wine gall.
268.8	1,000000	{,003720}			Malt gall.
2150.42	1,000000	{,000465}			Malt bush.
227.	1,000000	{,004405}			Mash tung.

And in like manner were all the other factors for squares found, which are inserted in the following table.

ART. IV.

To find gage points for squares.

THOSE gage points are no other than the square roots of the solid capacities of each gallon, bushel, &c. therefore extract the square root from the number of inches to a gallon or bushel, &c. and it is done. As for example :

The square roots of	{	282.	}	are	{	16.79	}	The square gage points for	{	Ale gallons.
		231.				15.19				Wine gallons.
		268.8				16.39				Malt gallons.
		2150.42				46.37				Malt bushels.
		227.				15.07				Mash tun gallons.

ART.

ART. V.

A Table of Factors.

	Factors for		Divisors for		Gage points for	
	Squares	Circles.	Squares	Circles.	Squares	Circles.
Inches the area of unity	1	,785398	1	1.27324	1	1.128
A superficial foot	,006944	,005454	144.	183.34	12.	13.54
A solid foot	,000578	,000454	1728.	2200.16	41.57	46.91
Ale gallon	,003546	,002785	282.	359.05	16.79	18.95
Wine gallon	,004329	,003399	231.	294.12	15.19	17.15
Malt or corn bushel	,000465	,000365	2150.42	2738.00	46.37	52.32
Malt gallon	,003720	,002922	268.8	342.24	16.39	18.5
Mash tun gallon	,004405	,00346	227.	289.	15.1	17.07
A pound of hard soap cold	,036845	,028939	27.14	34.56	5.21	5.88
A pound of hot soap	,035714	,028050	28.0	35.65	5.29	5.97
A pound of green soap	,038956	,0306	25.67	32.68	5.06	5.72
A pound of white soft soap	,039123	,030731	25.56	32.54	5.05	5.7
A pound of tallow net	,031844	,025101	31.4	39.98	5.6	6.32
A pound of green starch	,028736	,022565	34.8	44.32	5.9	6.66
A pound of dry starch	,024813	,019491	40.3	51.3	6.35	7.16
A pound of flint glass	,094697	,074405	10.56	13.44	3.25	3.69
A pound of white glass	,071123	,05586	14.06	17.9	3.74	4.32
A pound of green glass	,082102	,064516	12.18	15.5	3.48	3.94

Having found proper factors and divisors both for squares and circles, as in the above table, the area of any square or circular figure may be found in any of

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the denominations therein mentioned by the following general rules :

1. For a square, or an oblong figure.

Multiply the two sides of any square or oblong figure together, and their product multiply or divide by its proper factor or divisor for squares in the above table; then the last product or quotient will be the area of the same denomination as the factor, &c. made use of.

2. For circular figures.

Multiply or divide the square of the diameter of any circle, or the rectangle of the transverse and conjugate diameters of the ellipsis, by its proper factor or divisor in the above table, and the product or quotient will be the area of the same denomination as the factor, &c. made use of.



CHAP. VII.

Of finding the areas of superficies.

ART. 1.

To find the area of a geometric square, like the figure annexed.

Rule by pen.

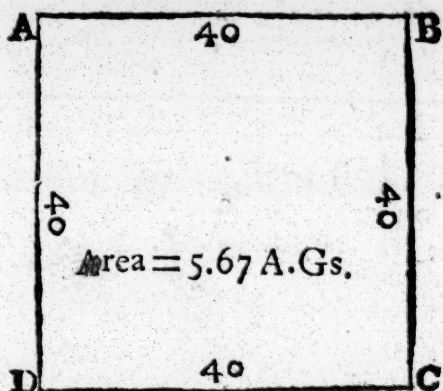
MULTIPLY one of the sides of the square by itself, and that product multiply or divide by the factor, &c. for squares in page 57, and the product or quotient will be equal to the area of the same kind as the factor or divisor made use of.

EXAMPLE.

Let the side of the square A B C D equal 40 inches, what is the area in ale gallons ?

SQUARE.

SQUARE.



OPERATION.

By Division.

$$\begin{array}{l} A B = 40 \\ B C = 40 \end{array}$$

$$282 \overline{) 1600} (5.67 = \text{Area.}$$

$$\underline{1410}$$

$$1900$$

$$\underline{1692}$$

$$2080$$

$$\underline{1974}$$

Rem. . 106

By Multiplication.

$$\text{Square Factor} = ,003546$$

$$\text{Square of } A B = 1600$$

$$\underline{2127600}$$

$$\underline{3546}$$

$$\text{Area} = 5,673600$$

I have wrought this example both by the divisor and factor for square ale gallons, and the area comes out the same, viz. 5.67 ale gallons both ways. But in my future operations I shall only make use of the divisor, because by that the area is found with the fewest figures in most cases. In like manner may the area of the square be found, in any of the other denominations mentioned in the table, by making use of the respective factors, &c. for if, instead of 282, I had divided by 231, 2150.42, the area would have been found in wine gallons,

60 Of finding the Areas of Superficies.

gallons, or malt bushels, and so of any of the rest; which is so plain, that I shall confine myself in all my operations to one of the denominations, which, if understood, any of the others may be performed in the same manner.

To find the same by the rule.

The proportion is,

on A.	on B.	on A.	on B.	
As 282	: 40	: : 40	: 5.67	= before found.
Sq. Divisor.	Side.	Side.	Area.	

Or thus,

Set unity on c to the square gage point on D, and against any side of a square on D is the area on c.

OPERATION.

on D.	on c.	on D.	on c.	
As 16.79	: 1	: : 40	: 5.67,	as before.
Sq. Gage pt.	Unity.	Side of Sq.	Area.	

The rule being thus set, it is a table; for against any side of a square upon D is the area upon c.

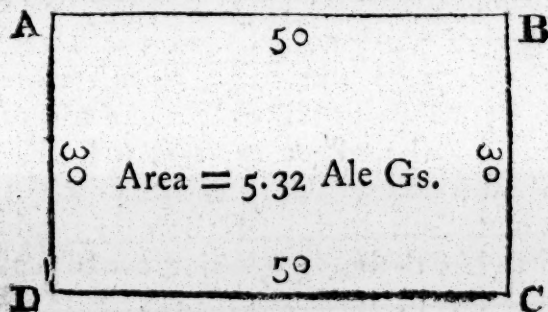
ART. II.

To find the Area of the Parallelogram A B C D.

Rule by pen.

MULTIPLY the longest by the shortest side, and that product multiply or divide by the factors or divisors in page 57, and the product or quotient will be the area required.

PARALLELOGRAM.



E X A M P L E

E X A M P L E.

Let the longest side of the parallelogram = 50 inches = A B, the shortest side = 30 inches = B C, what is the area of it in ale gallons?

O P E R A T I O N.

Longest side A B = 50

Shortest side B C = 30

Squ. divisor = 282) 1500 (5.32 fere the area in ale gall.

1410

..900

846

• 540

564

24 Too much.

The area above found is not quite 5.32; but it is nearer to 5.32 than to 5.31, and because in the practice of gaging we use but two decimal places of figures, therefore we take the nighest to the second place, whether it be more or less.

The proportion by the rule.

on A. on B. on A. on B.

As 282 : 50 : : 30 : 5.32 the same as before.

Sq.Divisor. Lon.Si. Sh.Si. Area.

A R T. III.

To find the area of a rhombus whose sides are equal and parallel, and two of its opposite angles are acute, and the other two are obtuse. See the figure following.

R U L E.

MULTIPLY the perpendicular or shortest distance between the two opposite sides by one of the sides, and that product multiply or divide by the factors,

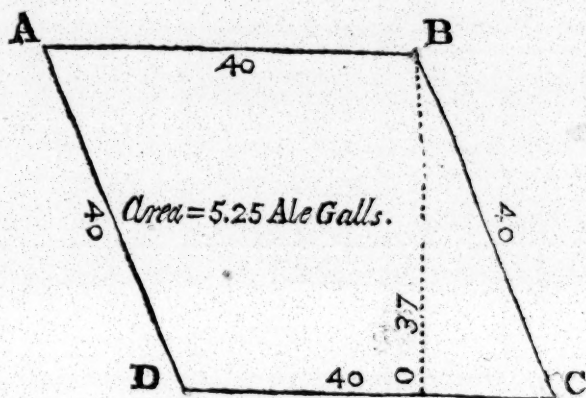
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factors, &c. for squares in page 57, and the product or quotient will be the area required.

EXAMPLE.

Let the side of the rhombus $A B C D = 40$ inches $= A B = B C$ and the perpendicular $B O = 37$ inches, what is its area in ale gallons?

R H O M B U S.



OPERATION.

Perpendicular ——— $B O = 37$

Side ——— $A B = 40$

Square div. for ale gall. $= 282$ $1480 (5.25 = \text{Area.})$

1410

$..700$

564

1360

1410

$..50$ Too much.

The proportion by the rule.

on A. on B. on A. on B.

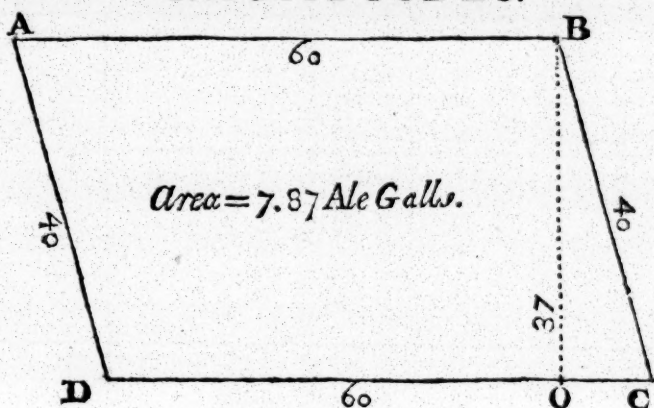
As $282 : 40 : : 37 : 5.25$ as before.

Sq. divisor. Side. Perpend. Area.

ART. IV.

To find the area of a rhomboides, whose two longest sides are equal and parallel, and the two shortest sides are also equal and parallel, two of its opposite angles are obtuse, and the other two acute. See the following figure.

RHOMBOIDES.



EXAMPLE.

LET the last figure represent the rhomboides, whose longest sides AB and $DC = 60$ inches, perpendicular $BO = 37$ inches; to find the area in ale gallons.

Operation by pen.

Perpendicular — — $BO = 37$
 Longest side — $AB = DC = 60$

Square ale divisor — — — — — $= 282)2220(7.87$ Ale area.

$$\begin{array}{r}
 1974 \\
 \hline
 2460 \\
 2256 \\
 \hline
 2040 \\
 1974 \\
 \hline
 \end{array}$$

Remains — .66

The proportion by the rule.

on A. on B. on A. on B.
 As 282 : 60 :: 37 : 7.87 = as before.
 Sq. div. Long. side. Perp. Area.

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ART. V.

To find the area of a plain triangle.

DEFINITION.

A Triangle consists of three sides and three angles : of which there are two sorts, the one a rectangled triangle, and the other an oblique angled triangle ; but as one general rule will serve for both sorts, I shall only give an example of an oblique angled triangle, which must be divided into two rectangled triangles, in order to find the area thereof.

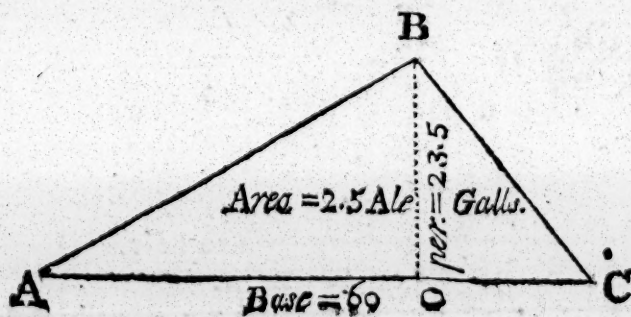
RULE.

From the greatest angle let fall a perpendicular on the longest side, and multiply half that perpendicular by the longest side, or half the longest side by the whole perpendicular, and that product multiply or divide by the factors &c. in page 57, and the product or quotient will be the area required.

EXAMPLE.

Let the longest side or base of the triangle $ABC = 60$ inches $= AC$, and the perpendicular $BO = 23.5$ inches ; to find its area in ale gallons.

TRIANGLE.



OPE.

OPERATION.

Perpendicular — B O = 23.5
 Half base ——— A C = 30

Square ale divisor = 282) 705,0 (2.5 = Area in ale gall.

564
 ———
 1410
 1410
 ———
 0

The proportion by the rule.

on A.	on B.	on A.	on B.
As 282	: 30	: 23.5	: 2.5 = as before.
Sq. div.	$\frac{1}{2}$ Base.	Perp.	Area.

ART. VI.

To find the area of a trapezium.

DEFINITION.

A Trapezium is composed of four sides and four angles; both the sides and the angles are unequal. See the following figure.

RULE.

Divide the trapezium into two triangles by a right line drawn from one angle to its opposite angle, which is called a diagonal, and divides the trapezium into two triangles, then let fall a perpendicular from each of the other angles upon that diagonal, which is a common base to both triangles.

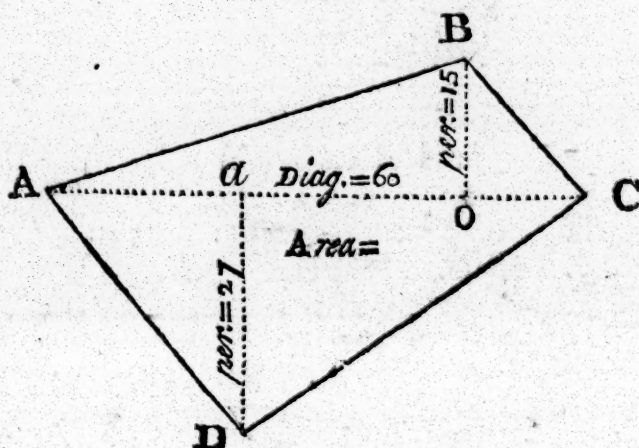
2. Multiply half the diagonal by the perpendiculars, or half the sum of the perpendiculars by the whole diagonal, and that product multiply or divide by the factors, &c. in page 57, and the product or quotient will be the area required.

EXAM-

EXAMPLE.

Let the figure $A B C D$ represent the trapezium, whose diagonal $A C = 60$ inches, the perpendicular $B O = 15$ inches, and the perpendicular $D a = 27$ inches : to find the area in ale gallons.

TRAPEZIUM.



OPERATION,

Perpendicular — $B O = 15$

Perpendicular — $D a = 27$

Sum = 42

$\frac{1}{2}$ Diagonal $A C = 30$

Square ale divisor = 282) 1260 (4.47 = Area in ale gall.

1128

1320

1128

1920

1974

54 Too much.

The

The proportion by the rule.

on A. on B. on A. on B.
 As 282 : 42 : : 30 : 4.47 = as before.
 Sq. div. Perpen. $\frac{1}{2}$ Diag. Area.

Note here, that the areas of the five last figures may be found by the lines c and d on the rule, the same as if they were squares, if there be first a mean geometrical proportional found between the perpendiculars and their bases; but the method of finding this mean proportional shall be the business of the next article.

ART. VII.

To find a mean geometrical proportional between two given numbers.

RULE.

MULTIPLY the two given numbers together, and extract the square root from their product, which will be the mean required.

EXAMPLE.

Let the given numbers be 72 and 50; to find a mean proportional between them.

OPERATION.

$$\begin{array}{r}
 72 \\
 50 \\
 \hline
 3600(60 = \text{Mean required.} \\
 30 \\
 \hline
 000
 \end{array}$$

By the rule.

Set one of the given numbers upon c to the same number upon d, then against the other given number upon c is the number sought upon d.

Thus,

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Thus,

on c.	on D.	on c.	on D.	
As 50	: 50	: : 72	: 60	= the same as before.
Given	Given	Given	Mean.	
numb.	numb.	numb.		

Another example by the rule.
Find a mean between 42 and 30.

OPERATION.

on c.	on D.	on c.	on D.	
As 42	: 42	: : 30	: 35.5	= Mean required.
Given	Given	Given	Mean.	
numb.	numb.	numb.		

This mean last found is a mean proportional between the sum of the perpendiculars and half of the diagonal of the trapezium, by which the area of that figure may be found by the lines c and D, according to the following proportion :

on D.	on c.	on D.	on c.	
As 16.79	: 1	: : 35.5	: 4.47	= Area found by
Sq. Gage	Unity.	Mean.	Area.	the lines A
point.				and B.

And in like manner may the area of the parallelogram, rhombus, rhomboides, and triangle be found.

ART. VIII.

To find the area of any regular polygon.

DEFINITION.

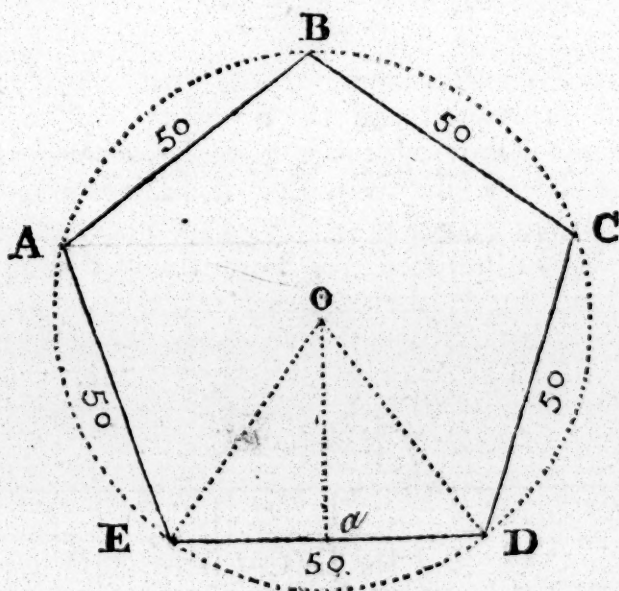
ALL figures that have more than four sides, and those sides equal, and so situated that a circle will pass through all its angles, are called regular polygons, and are named according to the number of sides they consist of, viz. if it has five sides it is called a pentagon,

if six fides a hexagon, if seven fides a heptagon, &c. as may be feen by the following table of polygons.

RULE.

From the center of the polygon let fall a perpendicular upon one of its fides, which will be the perpendicular of a triangle, and the fide will be the bafe; then find the area of that triangle as was fhewn in Art. V. of this chapter; and becaufe there are as many triangles as there are fides in the polygon, multiply the area of the triangle by the number of fides, and the product will be the area of the polygon required.

PENTAGON.



EXAMPLE.

Let the figure annexed represent a pentagon, whose fides $AB = BC = CD = DE = EA = 50$ inches; and the perpendicular $oa = 34.4$ inches, what is its area in ale gallons;

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OPERATION.

$$\text{Perpendicular } o a = 34,4$$

$$\frac{1}{4} \text{ of one of the sides} = 25$$

$$1720$$

$$688$$

$$\text{Sq. ale div.} = 282) \begin{array}{r} 860,0 \\ 846 \\ \hline \end{array} \quad \begin{array}{r} 3,0496 \\ 5 \\ \hline \end{array} = \text{Area of the trian. E o D.}$$

$$1400$$

$$1128$$

$$15.2480 = \text{Area of the pentagon in ale gallons.}$$

$$2720$$

$$2538$$

$$1820$$

$$1692$$

Remains 128

But the areas of any of the regular polygons mentioned in the following table may be found without knowing the perpendiculars, as will be shewn hereafter.

The table being made from a trigonometrical calculation, I have omitted to shew the method of its construction; for it would be of no use to a person who does not understand the rules of Plane Trigonometry, and such as do may easily find it out.

A Table of Regular Polygons.

Num. of Sides.	Names of the Polygons.	Area.	Area.	Area.	Area.
		Sq. Inches.	Ale Gs.	Wine Gs.	Malt Bs.
5	Pentagon	1.72	,006101	,007448	,00078
6	Hexagon	2.598	,009213	,011247	,001208
7	Heptagon	3.634	,012886	,015731	,001689
8	Octagon	4.828	,017122	,020902	,002245
9	Nonagon	6.182	,021921	,026761	,002875
10	Decagon	7.694	,027284	,033308	,003579
11	Undecagon	9.366	,033211	,040543	,004355
12	Duodecagon	11.196	,039631	,048468	,005205

The

Of finding the Areas of Superficies. 71

The use of the preceding table.

Multiply the square of the side of any regular polygon mentioned in the table by the common factor belonging to that polygon, and the product will be the area in inches, ale gallons, wine gallons, or malt bushels, respectively.

For example, Let the side of a pentagon be 50 inches, what is its area in ale gallons?

OPERATION.

Side of the pentagon = 50
 50
 —

Square of the side = 2500

The tabular number for ale gallons = ,006101

Square of the side — — = 2500

3050500
 12202
 ———

The area in ale gallons = 15.252500

Which is nearly the same as that found by letting fall the perpendicular.

In like manner may the area of any of the other regular polygons mentioned in the table be found in any of the denominations there mentioned; and if the area were required in any other denomination not there specified, it is only dividing the area of that polygon whose side is an inch, by the square inches contained in a foot, pound, or gallon, &c. and you will have a common factor for that denomination.

All other right-lined figures, consisting of more than four unequal sides, are called irregular polygons, and must be reduced into triangles by diagonal lines in the same manner as was done in the trapezium, Art. 6. of this chapter; then by the foregoing rules find the area of each triangle or trapezium which it is divided into, and the sum of the areas of the several parts will be equal to the area of the whole figure.

ART.

(72)

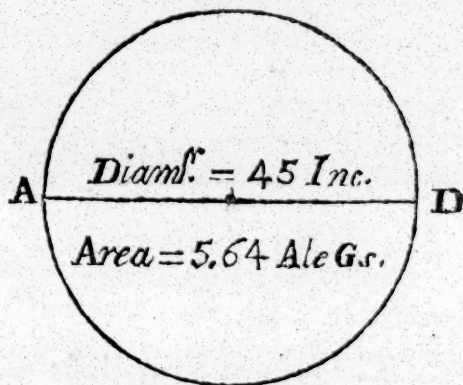
ART. IX.

To find the area of a circle.

RULE.

SQUARE the diameter of the circle, and that square multiply or divide by the factors, &c. in page 57 for circles, and the product, or quotients, will be the area required.

CIRCLE.



EXAMPLE.

Let the diameter A D of the above circle = 45 inches, what is the area of it in ale gallons?

OPERATION.

Diameter A D = 45

Diameter A D = 45

225

180

Circ. divisor = 359.05) 2025.00 (5.64 fere = Area in
1795.25 ale gall.

229750

215430

143200

143620

420

The

The proportion by the rule.

on D.	on C.	on D.	on C.	
As 18.95	: 1	:: 45	: 5.64	= as above.
Cir. Gage	Unity.	Diameter.	Area.	
point.				

The rule being thus set, it is like a table; for against any diameter on D is the area in ale gallons on C. So if unity on C was set to any of the other circular gage points on D, then against any diameter on D you will find the area on C, of the same denomination as the gage point that unity was set to.

ART. X.

To find the area of an ellipsis or oval.

DEFINITION.

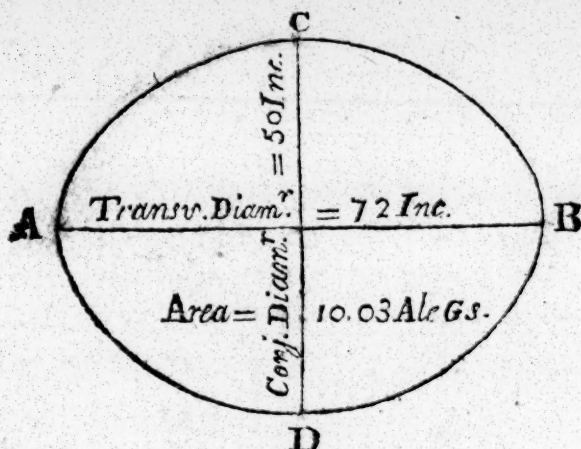
AN ellipsis is included in a curve line that is no part of a circle, but is generated by the section of a cone, cut obliquely through its axis, and is longer one way than the other.

The longest diameter A B is called the transverse diameter, and the shortest diameter C D is called the conjugate diameter; and it is a geometrical mean proportional between two circles whose diameters are equal to the transverse and conjugate diameters of the ellipsis.

RULE.

Multiply the transverse and conjugate diameters together, and that product multiply or divide by the factors, &c. in page 57 for circles; then that product or quotient will be the area of the ellipsis required.

ELLIPSIS.



EXAMPLE.

Let the transverse diameter A B of the above ellipsis = 72 inches, and the conjugate diameter c d = 50 inches, what is the area of it in ale gallons?

OPERATION.

Transverse diam. A B = 72
Conjugate diam. c d = 50

359,05) 3600.00 (10.03 fere = Area in
35905 ale gallons.

... 95000
107715

12715 Too much.

The proportion by the rule.

on A. on B. on A. on B.
As 359 : 72 :: 50 : 10.03
Cir.div. Tr.diam. Cong.diam. Area.

Or thus,

Find a mean between the transverse and conjugate diameter, which, in this case, is = 60; then the proportion will be the same as that for a circle.

As

on D.	on C.	on D.	on C.	
As 18.95	: 1	: : 60	: 10.03	= as before.
Cir. Gage	Unity.	Mean.	Area.	
point.				

CHAP. VIII.

Of finding the contents of solids.

AS superficies are comprehended under length and breadth, so solids are comprehended under length, breadth, and depth; and though it be improper, in a geometrical sense, to ascribe thickness to a superficies, yet gagers always consider them as one inch deep, and accordingly compute the superficial content or area in ale gallons, &c. in solid measure.

Now by having the area or content given at one inch deep, it will be an easy matter to find the content of any solid body at any depth, provided the bases of that body are equal, and straight lines may be every where applied from one base to the other; for if the area or content at one-inch deep be multiplied by the whole depth, the product will be the solid content of the body.

From hence it will be very easy to find the solidity of any body that hath any of the figures in the last chapter for its bases, which figures I shall here make use of for the bases, supposing them to have depth, as well as length and breadth.

ART. I.

To find the content of a solid whose bases are either squares or parallelograms, both equal and parallel to each other, and of equal substance through every part of the depth.

RULE.

MULTIPLY the length of the base by the breadth, and that product by the depth, and this last product multiply or divide by the factors, &c. for squares

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in page 57, and the product or quotient will be the content required, of the same kind as the factor, &c. made use of.

Or,

Find the area of the base by Art. 1, and 2, of the last chapter, and multiply that area by the depth, and the product will be the content.

Example 1. for a square base.

Let the figure in Art. 1. of the last chapter represent the base of the solid whose side is = 40 inches, and suppose its depth = 10 inches, what will its content be in ale gallons?

OPERATION.

$$\begin{array}{r}
 40 \\
 40 \\
 \hline
 \text{Square of the side} = 1600 \\
 \text{Depth} \quad \quad \quad = \quad 10 \\
 \hline
 \text{Square divisor} = 282 \overline{)16000} (56.7 = \text{Content in ale gal-} \\
 \quad \quad \quad 1410 \quad \quad \quad \text{lons.} \\
 \hline
 \quad \quad \quad 1900 \\
 \quad \quad \quad 1692 \\
 \hline
 \quad \quad \quad 2080 \\
 \quad \quad \quad 1974 \\
 \hline
 \text{Remains} \quad .106
 \end{array}$$

The area of the base of this solid was found in Art. 1. of the last chapter to be = 5.67, which multiplied by 10, the depth, = 56.7 = the content in ale gallons, the same as above found.

The proportion by the rule.

on D.	on c.	on D.	on c.	
As 16.79	: 10	: :	40	: 56.7 = as before.
Sq. Gage	Depth.	Side of	Content.	
point.		square.		

Example

Example II. for an oblong base.

Let the figure in Art. 2. of the last chapter represent the base of the oblong solid, whose longest side $AB = 50$ inches, shortest side $AD = 30$ inches, and suppose the depth = 10 inches, what is its content in ale gallons?

OPERATION.

Side A B	—	= 50
Side A D	—	= 30
	—	
	1500	
Depth	=	10
	—	
Square divisor = 282	15000	53.2 = Content in ale
	1410	gallons.
	—	
	..900	
	846	
	—	
	.540	
	564	
	—	

24 Too much.

The area of the base was found in Art. 2. of the last chapter to be = 5,32, which multiplied by 10 = 53.2, the content in ale gallons, the same as that before found.

The proportion by the rule is the same as that for a square base, when there is first a mean proportional found between the longest and shortest sides of the base; as for example:

1 For the mean proportional.

on c.	on D.	on c.	on D.	
As 50	:	50	:	30 : 38.73 = Mean sought.
L. side.	L. side.	S. side.	Mean.	

2. For the content.

on D.	on c.	on D.	on c.	
As 16.79	:	10	:	38.73 : 53.2 = as by pen.
S. Gage pt.	Depth.	Mean.	Content.	

E 3

Or.

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Or the content may be found by the lines A and B, by first finding the area of the base, as in Art. 2. of the last chapter, and then multiplying that area by the depth; which product will be the content. This is so plain, that it requires no examples.

In like manner may the content of any right-lined solid be found, (whether it has a triangle, a rhombus, or rhomboides; or if it has any of the polygons, either regular or irregular, for its bases) if by the foregoing rules you find the area of its base, and multiply that area by its depth.

A R T. II

To find the content of a solid that has two equal circles for its bases, commonly called a cylinder, or if it has two equal ellipsis for its bases called a cylindroid, by having the depth and diameters given. It is supposed that the bases are parallel, and that the bodies are of an equal diameter throughout the whole depth.

R U L E.

BY Art. 9 and 10. of the last chapter find the areas of the bases, which areas multiply by the depth, and the product will be the content of the solid required.

Example I. for the cylinder.

Let the figure in page 72 represent the bases of the cylinder, whose diameter is = 45 inches = A. B, and suppose the depth = 12 inches; to find the content in ale gallons.

O P E R A T I O N.

The area of the base, is found by Art. 9, }	= 5.64
chap. vii. ——— ——— ——— }	
Depth ——— ——— ———	= 12
	————
	1128
	564
	————

Content in ale gallons = 67.68
The

The Proportion by the rule.

on D.	on C.	on D.	on C.	
As 18.95	: 12	:: 45	: 67.68	= as before.
C. Gage pt.	Depth.	Diam.	Content.	

Example II. for a cylindroid.

Let the figure in page 74 represent the bases of the cylindroid, whose transverse diameter $A C = 50$ inches, and suppose the depth to be $= 12$ inches, how many gallons will it contain?

OPERATION.

The area of the base as found in Art. 10,	}	= 10.03
chap. vii.		
Depth		= 12
		2006
		1003

The content in ale gallons $= 120, 36$

The proportion by the rule is the same as for a cylinder, when first there is a mean proportional found between the transverse and conjugate diameters.

1. For the mean proportional.

on C.	on D.	on C.	on D.	
As 72	: 72	:: 50	: 60	= the mean diam.
Tr. diam.	Tr. diam.	Conj. dia.	Mean.	

2. For the content.

on D.	on C.	on D.	on C.	
As 18.95	: 12	:: 60	: 120.36	= as before.
Circ. gage	Depth.	Mean.	Content.	

point.

Or the content may be found by the lines A and B at two operations, viz. first find the area, and then multiply that area into the depth, and the product will be the content. As for example:

1. For the area,

on A.	on B.	on A.	on B.	
As 359	: 72	:: 50	: 10.03	= Area.
Cir. Div.	Tr. Diam.	C. diam.	Area.	

E 4

2. For

2. For the content.

on B.	on A.	on B.	on A.	
As 1	: 10.03	: 12	: 120.36	= Content before
Unity.	Area.	Depth.	Content.	found.

In like manner may the content be found in any other denomination, by making use of the respective factor, &c. for that denomination.

But the area or content of any square or circular figure may be found at once setting the rule for each, in all the different denominations, by taking out the slide on which the line c is, and putting it in again the reverse or backward way, and counting the numbers in that order; for then if unity on c be set to the side of a square, or diameter, or mean side of a parallelogram, or mean diameter of an ellipsis, against any gage point on D is the area on c. Or if the depth on c be set to the side of a square, or diameter, &c. on D, then against any gage point on D is the content on c. As for example, let the side of a square or diameter = 30 inches, set unity on c to 30 on D.

then against the sq. gage pts. on D for	{	Ale gs. Wine gs. Malt bs.	}	are on c	{	Areas. 3.18 3.90 0.42	}	and against the cir. gage pts. on D are	{	Areas. 2.51 3.06 0.33	}	on c.

Suppose the depth = 8 inches, then set 8 on c to 30 on D.

then against the sq. gage pts. on D for	{	Ale gs. Wine gs. Malt bs.	}	are on c.	{	Contents. 25.44 31.20 3.36	}	and against the cir. gage pts. on D are	{	Contents. 20.08 24.48 2.64	}	on c.

ART.

A R T. III.

To find the content of a pyramid or cone.

D E F I N I T I O N.

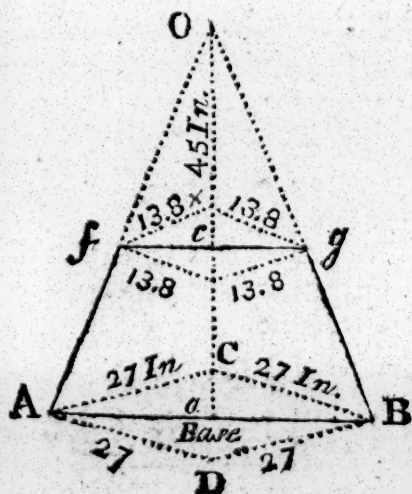
A Pyramid is a solid whose base is a polygon, and its sides are triangles, whose tops all meet in a point called its vertex.

A cone hath a circular base, and tapereth away to a point called its vertex, in such a manner that all lines along the slant surface are straight lines; the pyramid and cone being so nearly related to each other, that one general rule may suffice for finding the solid contents of both, for either of them are equal to one third part of a solid of the same height, and of the same area at the base.

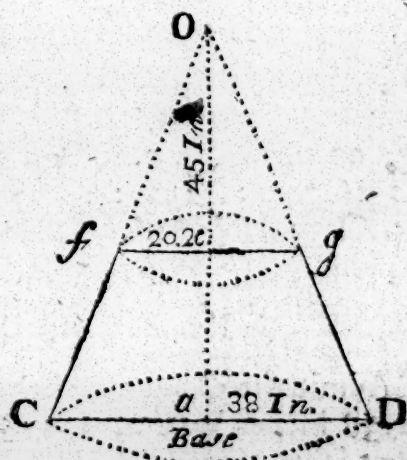
R U L E.

Find the area of the base by the foregoing rules, whether it be a circle, square, or any of the regular polygons; which area multiply by one third part of its height; or, which is the same, multiply the whole height by one third of the area of its base, and the product will be the content required.

A SQUARE PYRAMID.



A CONE



Of finding the Contents of Solids.

Example I. For the square pyramid.

Admit the side of the base of the pyramid $A O B = 27$ inches $= A C = C B = B D = D A$, and the altitude $O a = 45$ inches, what is its content in ale gallons?

OPERATION.

Side of the base $= A C = 27$

Side of the base $= C B = 27$

—
189

54
—

Square ale divis. $= 282)729($

564
—

1650

1410
—

2400

2256
—

• 1440

1410
—

Remains — .. 30

To find the same by the rule.

PROPORTION.

on D.	on C.	on D.	on C.
As 16.79	: 15	: : 27	: 38.77 as above.
Sq. Gage	$\frac{1}{3}$ Alti.	Side of	Content.
point.	tude.	Base.	

Example II. For the cone.

Admit the diameter of the base of the cone $C O D$ to be 38 inches $= C D$, and its altitude 45 inches $= O a$, what is its content in ale gallons?

OPE.

OPERATION.

The diameter c D = 38

The diameter c D = 38

$$\begin{array}{r} 304 \\ 114 \\ \hline \end{array}$$

Circ. div. = 359.05) 1444,00(

$$\begin{array}{r} 143620 \\ \hline \end{array}$$

4.022 = area.

15 = $\frac{1}{3}$ C a.
$$\begin{array}{r} .78000 \\ 71810 \\ \hline \end{array}$$

20110

4022

$$\begin{array}{r} .61900 \\ 71810 \\ \hline \end{array}$$
60,330 = cont. in
ale gs.

.9910 Too much.

To find the same by the rule.

PROPORTION.

on D.	on C.	on D.	on C.
As 18.95	: 15	: 38	: 60.3 = nearly as above.
C. gage	$\frac{1}{3}$ Altitude.	Diam.	Content.
point.			

ART. IV.

To find the content of the frustum of a pyramid or cone.

DEFINITION.

A Frustum of a pyramid or a cone, is the lower part or remainder of a pyramid or cone, when the small ends are cut off, as A f g B is a frustum of the square pyramid, and c f g D is a frustum

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of the cone: see the figures in the last article. And as a pyramid and cone are nearly related to each other, so are also their frustums. Therefore one general rule will serve for finding the contents of both.

R U L E.

To three times the rectangle of the greatest and least sides of the bases of the pyramid's frustum, or the greatest and least diameter of the cone's frustum, add the square of the difference of the sides of the pyramid, or the difference of diameters of the cone's frustum bases, and this sum multiply by one third of the height, and multiply or divide that product by the factors, &c. for squares or circles in page 57; then the product or quotient will give the content required.

Example I. For a frustum of a square pyramid.

Let the lower part of the pyramid $AfgB$ represent the frustum, whose side of the greater base $Ac = 27$ inches, and the side of the lesser base $fx = 13.8$ inches, and altitude or depth $ac = 21$ inches; to find its content in ale gallons.

OPERATION.

Side of the lesser base $fx = 13.8$	27.
Side of the greater base $Ac = 27.$	13.8
	966
	276
Rectan. of the sides =	372.6
	3
3 Times the rectang. } of the sides —	1117.8
Sq. of the diff. of sides =	174.24
	1292.04
$\frac{1}{3}$ of the Altitude $ac =$	7
Product — — =	9044.28

Square

Squ

two
Ar
the
to

tha
for
of

fru
inc
inc
fin

Of finding the Contents of Solids. 85

Square divis. = $282)9044.28(32.07 = \text{cont. in ale gs.}$
 $\begin{array}{r} 846 \\ \hline \end{array}$

$\begin{array}{r} .584 \\ 564 \\ \hline .2028 \\ 1974 \\ \hline \end{array}$

Remains — ..54

To find the same by the rule.

By Art. vii. Chap. vii. find a mean proportion between the sides of the greater and lesser bases; then by Art. i. Chap. vii. find the areas of those three squares, the sum of which multiplied by $\frac{1}{3}$ of the depth, is equal to the content of the frustum.

OPERATION.

Set unity on c to 16.79 on D, then against the

	on D.		on c.
{	Side of the greater base = 27	} are	2.58
	Side of the lesser base = 13.8		those { 0.676
	Side of the mean base = 19.3		areas { 1.32

$\begin{array}{r} \text{Sum of the areas} \quad \text{—} \quad = 4.576 \\ \frac{1}{3} \text{ of the depth } ac \quad \text{—} \quad = \quad 7 \\ \hline \end{array}$

Content in ale gallons = 32.032

The content found by the rule agrees very well with that found by the pen. And as this method will serve for either pen or rule, it may be used for the pen instead of the former rule.

Example II. For the frustum of a cone.

Let the lower part of the cone $cf g D$ represent the frustum whose diameter at the greater base $c D = 38$ inches, and the diameter at the lesser base $f g = 20.2$ inches, and the altitude or depth $ac = 21$ inches; to find its content in ale gallons.

OPE-

OPERATION.

The lesser diam. $fg = 20.2$ Greater diam. $cd = 38.$ 1616606767.633 Times rect. of }
the diameters. — } = 2302.8Square of diff. — = 316.84

Sum — — = 2619.64

 $\frac{1}{3}$ of the depth — = 7Cir. divis. = 359.05) 18337.48 (51.07 = Cont. in ale
gallons.179525.. 3849835905. 259300251335

Remains — 7965

To find the same by the rule. First find a mean proportional between the greater and lesser diameter, and then the work will stand thus.

OPERATION.

Set unity on c to 18.95 on d , then against the

	on d	on c .	
{	Diam. of the greater base = 38	{ are } 4.02	
	Diam. of the lesser base = 20.2		{ those } 1.14
	Mean diameter — = 27.7		{ areas } 2.14

Sum of the areas — = 7.30

 $\frac{1}{3}$ of the depth ac — = 7The content in ale gallons — = 51.10
This

This also agrees very well with the content before found, which method will serve for both pen and rule.

The answer would have been the same if I had not found a mean diameter, but instead thereof found a mean area between the areas of the greater and lesser diameters, and added the three areas together as above, and multiplied their sum by $\frac{1}{3}$ of the depth, the product would give the content the same as that before found. This last method is universal for all frustums of pyramids or cones, let their bases be of what form they will, whether they be parallelograms or ellipses, provided their bases are parallel, and there be the same proportion between the longest and shortest sides, or diameters of the greatest base, as there is between the longest and shortest sides or diameters of the lesser bases, and where straight lines may be applied from base to base throughout every part of the depth.

ART. V.

To find the content of a solid whose parallel bases are unequal, (the one base may be a square, the other a parallelogram, or the one an ellipsis and the other a circle) the sides being straight from top to bottom, whether the bases are alike or not alike, proportional or disproportional, alike situated or inverted.

General Rule.

1. **T**O the length, or transverse diameter of the greater base, add half the length, or half the transverse diameter of the lesser base, and multiply this sum by the breadth or conjugate diameter of the greater base; the product reserve.
2. To the length, or transverse diameter of the lesser base, add half the length, or half the transverse diameter of the greater base, and multiply this sum by the breadth or conjugate diameter of the lesser base, and add the product to the former reserved product, which sum multiply by one third of the depth, and multiply or divide the last product by the proper factors, &c. in

page 57, for squares or circles, and the product or quotient will be the content required, of the same kind as the factor, &c. made use of.

EXAMPLE.

Suppose a solid, whose bases are parallelograms or ellipses, and the longest side or transverse diameter of the greater base = 75 inches, and the shortest side or conjugate diameter = 50 inches; the longest side or transverse diameter of the lesser base = 45 inches, and the shortest side or conjugate diameter = 35 inches, and its depth = 30 inches; to find the content in ale gallons.

1. For the Square, &c. Bases.

OPERATION

Longest side, &c. } of greater base	= 75	Longest side of } greater base	= 45
$\frac{1}{2}$ longest side of } the lesser base	= 22.5	$\frac{1}{2}$ longest side of } greater base	= 37.5
Sum — —	= 97.5		82.5
Breadth of the } greater base	= 50	Breadth of the } lesser base	= 35
1 product — —	= 4875.0		4125
2 product — —	= 2887.5		2475
Sum — —	= 7762.5		2887.5
$\frac{1}{3}$ of depth — —	= 10		

Sq. div. = 282(77625.0(275.3 = Cont. in ale ga

564
2122
1974
1485
1410
750
846

~~Remains~~

— .96 Too much

2. For

-2. For the oval or elliptical bases.

OPERATION.

Cir. divisor = 359.05) 77625.00 (216.2 = Cont. in ale
gallons.

$$\begin{array}{r}
 71810 \\
 \hline
 .58150 \\
 35905 \\
 \hline
 222450 \\
 215430 \\
 \hline
 ..70200 \\
 71810 \\
 \hline
 \end{array}$$

.1610 Too much.

The operation is the same both for square and oval bases, till you come to make use of the factors, &c. in 57, as appears by the above work; where I find that if the bases are parallelograms, the content will be 275.3 ale gallons, but if ellipses 216.2 ale gallons.

ART. VI.

To find the content of a sphere or globe.

DEFINITION.

A Sphere or globe is two third parts of a cylinder, whose diameter and height is equal to the diameter of the sphere; therefore if the cube of the diameter of a sphere be multiplied by two third parts of the circular factors, &c. in page 57, or if it be divided by one, and an half of the circular divisors, then the product or quotient will be equal to the content of that sphere. From hence it will be easy to find factors and divisors for the cube of the diameter of any sphere for all the different denominations in page 57. As for example.

1st,

1st. For factors.

Factors for cylinder.

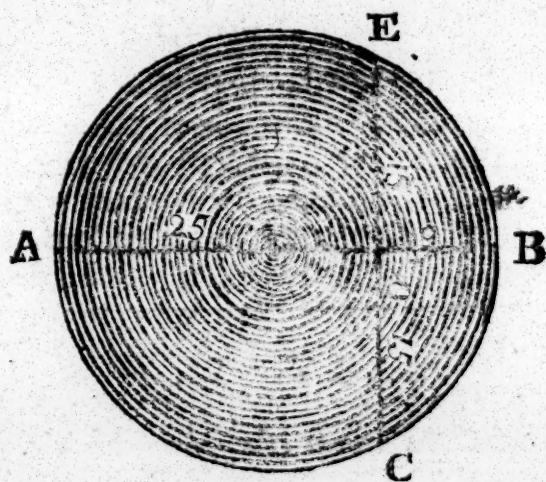
Factors for a sphere.

$$\frac{2}{3} \text{ of } \left\{ \begin{array}{l} .000365 \\ .002785 \\ .003399 \end{array} \right\} \text{ is equal } \left\{ \begin{array}{l} ,000243 \\ ,0018567 \\ ,002267 \end{array} \right\} \begin{array}{l} \text{Malt bs.} \\ \text{Ale gs.} \\ \text{Wine gs.} \end{array}$$

2. For divisors the proportion is

$$\text{as } \left\{ \begin{array}{l} 1 : 1,5 :: 2738. : 4107. \\ 1 : 1,5 :: 359.05 : 538.58 \\ 1 : 1,5 :: 294.12 : 441.18 \end{array} \right\} \text{ Div. } \left\{ \begin{array}{l} \text{M. bs.} \\ \text{A. gs.} \\ \text{W. gs.} \end{array} \right.$$

And in this manner may factors and divisors be found for a sphere in all the different denominations mentioned in page 57.



R U L E.

Cube the diameter of the sphere, and multiply or divide that cube by the factors, &c. above found, and the product or quotient will be the content of the sphere.

E X A M P L E.

Suppose the diameter of the sphere A E B C were 34 inches = A B, what will its content be in ale gallons? Vide last figure.

OPERATION.

	Content.
$A B = 34$	$538.58)39304.00(72.9 = A c$
34	377006
$\underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$
136	$.160340$
102	107716
$\underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$
Sq. of $A B = 1156$	$.526240$
34	484722
$\underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$
4624	Rem. — $.41518$
3468	
$\underline{\hspace{1cm}}$	

Cube of $A B = 39304$ The proportion by the lines D and E on the rule.on D . on E . on D . on E .

As 1 : ,001856 :: 34 : 72.9 = as above.

Unity. Factor. Diam. Content.

The rule being thus set, is a table, for against any diameter on D is the content in $A. G.$ on E ; the like for $W. G.$ or $M. B.$ for unity on D being set to the respective factor on E , then against any diameter on D is the content in $W. G.$ or $M. B.$ upon E .

The same may be performed by the lines c and D on the rule; for if the square roots of the several divisors be extracted, these roots will be the gage points for a sphere.

Thus the $\left\{ \begin{array}{l} 4107. \\ 538.58 \\ 441.18 \end{array} \right\}$ is $\left\{ \begin{array}{l} 64.1 \\ 23.2 \\ 21.0 \end{array} \right\}$ the $\left\{ \begin{array}{l} M. B. \\ A. G. \\ W. G. \end{array} \right\}$
 sq. root of gage pt. for

And the proportion will be

As the gage point on D is to the diameter of the sphere on c , so is the diameter on D to the content upon c . As for example in the above case.

on D . on c . on D . on c .As 23.2 : 34 :: 34 : 72.9 $A. G.$ as before.

Gage pt. Diam. Diam. Content.

In

In like manner may the content be found in any other denomination, provided you make use of its proper gage point.

ART. VII.

Having the altitude of the frustum of a sphere B O, with the diameter of the base E C; to find the altitude of the other frustum A O, or the remainder of the sphere's diameter. See the last figure.

RULE.

DIVIDE the square of half the diameter of the frustum's base by the altitude of the given frustum, and the quotient will be the altitude of the other frustum, or remainder of the sphere's diameter.

EXAMPLE.

Let the altitude of the frustum of the sphere E B C O = 9 inches = B O, and the diameter of the base E C = 30 inches; to find the altitude of the other frustum, or remainder of the diameter of the whole sphere.

OPERATION.

$$\frac{1}{2} EC = EO = 15$$

$$\begin{array}{r}
 15 \\
 \hline
 75 \\
 15 \\
 \hline
 9 \overline{)225} (25 = \text{alt. of other frustum} = AO. \\
 \underline{18} \\
 45 \\
 \underline{45} \\
 0
 \end{array}$$

ART. VIII.

To find the content of the lesser frustum of a sphere by having its diameter EC , and altitude BO given. See the last figure.

RULE.

TO three times the square of half the diameter of its base add the square of its altitude; this sum multiply by its altitude, and the product multiply or divide by the proper factors, &c. for a sphere in page 90, and the product or quotient will be the content required.

EXAMPLE

Let the dimensions be the same as in the last article, viz. diameter of the base $EC = 30$ inches, and the altitude $BO = 9$ inches; to find the content in ale gallons.

OPERATION.

Square of $\frac{1}{2}$ the diameter $EC = 225$	$BO = 9$
<u>3</u>	<u>9</u>
3 Times sq. of $\frac{1}{2}$ the diam. = 675	Sq. $BO = 81$
Square of altitude — = 81	
<u> </u>	
Sum — — — = 756	
Altitude BO — — = 9	
<u> </u>	

Content.

$$538.58)6804.00(12.6 = A. \text{ gs.}$$

$$\underline{53858}$$

$$141820$$

$$\underline{107716}$$

$$.341040$$

$$\underline{323148}$$

$$\text{Remains --- } .17892$$

ART.

A R T. IX.

To find the content of the greater frustum of a sphere, or the part C A E. See the last figure.

THIS may be found the same way as the lesser frustum was in the last article, or by the following rule, which will also serve for either frustum.

R U L E.

Multiply the square of half the diameter by three times the altitude of the frustum, and to this sum add the cube of the altitude; and this last sum multiply or divide by the factors, &c. for a sphere in page 90, the product or quotient will be the content required.

E X A M P L E

Let the diameter of the base $EC = 30$ inches, and the altitude $AO = 25$ inches: to find the content in ale gallons.

O P E R A T I O N.

	Alt. $AO = 25$
	25
	—
	125
	50
	—
Square of $\frac{1}{2}$ the diam. $EC = 225$	Sq. alt. = 625
3 Times the altitude $AO = 75$	25
—	—
1125	3125
1575	1250
—	—
Product — = 16875	C. alt. = 15625
Cube of the alt. — = 15625	
—	
538.58)32500.00(60.3 = Cont. in	ale gallons
323148	
—	
..185200	
161574	
—	
Remains — .23626	

P R O O F

P R O O F.

The content of the greater frustum C A E	=	60.3
The content of the lesser frustum E B C	—	12.6
		—
The content of the whole sphere	—	72.9

Which agrees with the content of the whole sphere, as found in Art. vi. of this chapter; and in like manner may the content be found in any other denomination mentioned in page 57, if only you reduce the factors, &c. for cylinders to those for a sphere, according to Art. vi. of this chapter.

A R T. X.

Having the top and bottom diameters of the frustum of a cone, to find intermediate diameters at any assigned depth.

R U L E.

DIVIDE the difference between the top and bottom diameters by the frustum's depth, and the quotient will be a common factor to multiply any depth by; the product of which added to the diameter of the lesser base (the depth was taken from the lesser base) will be equal to the diameter at that depth or distance from the base. But if the depth be taken from the greater base, then subtract the product of the common factor, and both from the diameter of the greater base, and the remainder will be equal to the diameter at the depth.

E X A M P L E

Let the cone's frustum be the same as that in Art. iv. of this chapter, where the diameter of the greater base = 38 inches, and the diameter of the lesser base = 20.2 inches, and the depth ac = 21 inches; to find intermediate diameters at every three inches of its height.

O P E-

OPERATION.

$$c d = 38$$

$$f g = 20.2$$

$$a c = 21 \overline{) 17.8} (.848 = \text{Com. factor.}$$

$$168$$

$$100$$

$$84$$

$$160$$

$$168$$

8 Too much.

Factor. Diff. Les. Diam. Diam.

$3 \times ,848 = 2.5 + 20.2 = 22.7$	} at these dists. from lesser base	{	3
$6 \times ,848 = 5.1 + 20.2 = 25.3$			6
$9 \times ,848 = 7.6 + 20.2 = 27.8$			9
$12 \times ,848 = 10.2 + 20.2 = 30.4$			12
$15 \times ,848 = 12.7 + 20.2 = 32.9$			15
$18 \times ,848 = 15.3 + 20.2 = 35.5$			18

If the differences above had been substracted from the greater diameter, the remainders would have been the diameters at the same distances from the greater base.

CHAP. IX.

Of gaging and fixing victuallers open utensils, &c.

IN this chapter I shall shew the method made use of by the officers of excise in gaging and finding the areas of victuallers open utensils, with the manner of fixing them in the dimension book; and also the method of gaging all sorts of by-tubs, &c. I shall first begin with the mash tun, and so proceed in the same order as they must stand in the dimension book.

ART

A R T. I.

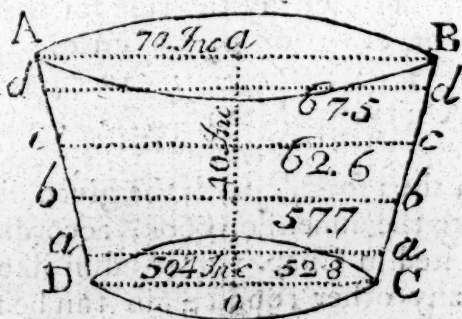
To gage and fix a viſtualler's maſh tun in form of the fruſtum of a cone, commonly called a round maſh tun. See the figure below.

R U L E.

1. **W**ITH a dimension cane, or Branen's rule, find the top and bottom diameters, and alſo the depth of the tun; then add the two diameters together, and take half that ſum for a mean diameter, which will be near enough the truth in practice.

2. To find the area at one inch deep, ſquare the mean diameter, and divide that ſum by the circular di- viſor for a maſh tun gallon in page 57, and the quo- tient will be the area at one inch deep, which ſerves as a mean for the whole depth.

A ROUND MASH TUN.



E X A M P L E.

Let the above figure represent the maſh tun, whoſe top diameter A B = 70 inches, the bottom diameter D C = 50.4 inches, and depth a o = 40 inches, as found by the dimension cane: to find the area at one inch deep.

OPERATION.

Top diameter A B — = 70

Bottom diameter p c = 50.4

Sum — — = 120.4

Half sum — — = 60.2 = mean diam.

60.2

120.4

36.120

Circular divisor = 289) 3624.04) 12.54 = area in gs.

289

.734

578

1560

1445

.1154

1156

...2

Too much.

The proportion by the rule for the area.

on D.

on C.

on D.

on C.

As 17 : 1 :: 60.2 : 12.54 = as above.

Circular Unity. Mean Area,

gag pt.

diam.

The area thus found, with the given dimensions, are to be put into the dimension book, according to the specimen exhibited in Chap. xiii. And in like manner may the area of any other round mash tun be found.

ART. II.

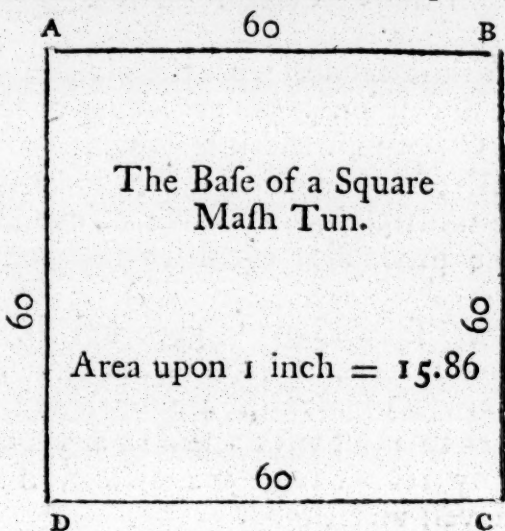
To gage and fix a mash tun, whose bases are equal square or oblongs, and parallel to each other, and the sides everywhere straight from base to base.

RULE.

1. **W**ITH some convenient instrument measure the length and breadth of the base, and also the depth of the tun,

2. Mul-

2. Multiply the length by the breadth of the base, and the product multiply or divide by the factors, &c. for a square mash tun in page 57, and the product or quotient will be the area at one inch deep.



EXAMPLE.

Let the above figure represent the base of a square mash tun, whose sides $AB = BC = CD = DA$, suppose I found with my dimension cane = 60 inches, and the depth = 30 inches; to find the area in mash tun gallons.

OPERATION.

Side $AB = 60$

Side $BC = 60$

Sq. divisor = 227)3600(15.86 = area in ale gallons.

$$\begin{array}{r}
 227 \\
 \hline
 1330 \\
 1135 \\
 \hline
 1950 \\
 1816 \\
 \hline
 1340 \\
 1362 \\
 \hline
 \dots 22
 \end{array}$$

Too much

F 2

The

The proportion by the rule.

on D.	on c.	on D.	on c.	
As 15.1	: 1	: 60	: 15.86	= as above.
Sq.gage	Unity.	Side of	Area.	
point.		sq.		

The area thus found, with the given dimensions, is put into the dimension book according to the specimen exhibited in Chap. xiii. and it is done.

N. B. All gages that are taken in mash tuns are cast up by multiplying the area of one inch by the depth, whose product is equal to the content in ale gallons.

Thus I have shewn how to gage and fix both round and square mash tuns, which are the most usual forms that these kinds of utensils are made in; but if you should chance to meet with a tun of any other form, it will be an easy matter to gage and fix it, if the foregoing rules be well understood.

ART. III.

To gage and fix a copper. See the following figure.

RULE.

1. **O**BERVE whether or no the copper hath any considerable fall in the middle, or if it hath a rising crown: and if it hath neither of them very considerable, then take the depth of it in the middle, between the center and side, as at E F.

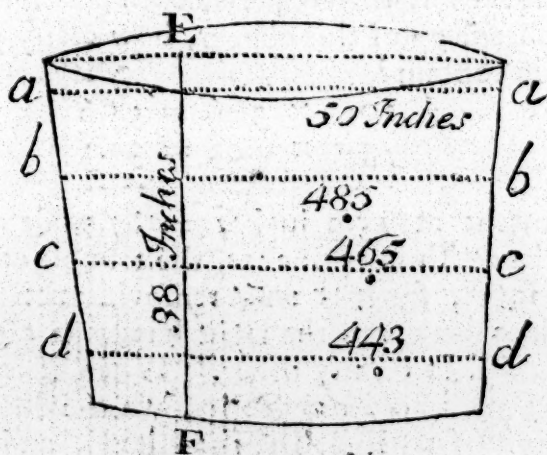
2. Take mean diameters between every six or ten inches. Thus on your Branen's rule or dimension cane, make marks with chalk at 3, 9, 15, or at 5, 15, 25 inches, according to the depth of the copper; then set the instrument perpendicular to the bottom of the copper, and at those several marks on the instrument mark also the sides of the copper on each opposite side, and also at cross diameters to the two former marks.

3. With

3. With your instrument take mean diameters at those marks, by turning it round at those places, in order to see if the diameters are equal every way, or not, which mean diameters enter on a piece of paper as you take them; as also the depth of the copper.

4. The next thing is to find the area at the several mean diameters, which may be done by Art. ix. of Chap. vii. or by the table of ale areas. The latter method I think is preferable for fixing of utensils, as it saves the trouble of finding the area by the pen, and you are sure to have the decimal true to two places of figures which you cannot be sure to have by the sliding rule, although you may have a good one. Nevertheless it will be proper to try them by the rule, in order to prevent any mistake that may happen in taking them out of the tables.

A COPPER.



EXAMPLE.

Let the above figure represent the copper to be gaged and fixed, and suppose that I found the dimensions of it by the directions in this article, as follows, viz. the depth = $EF = 38$ inches, the first mean diameter 5 inches from the bottom = $dd = 44.3$ inches, the second at 15 inches deep = $cc = 46.5$ inches, the third at 25 inches deep = $bb = 48.5$ inches, and the

fourth at 35 inches deep = $a a = 50$ inches; which mean diameters I set down as under, and turn to the table of ale areas, and take out the areas answering each respective diameter, which I set against those diameters, and then transfer the whole into the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

	Diam.	Areas.	
Mean.	$\{ a a = 50$	$= 6.96$	} as found by the table of ale areas.
	$b b = 48.5$	$= 6.55$	
Diam.	$c c = 46.5$	$= 6.02$	
	$d d = 44.3$	$= 5.46$	

The lower area 5.46 serves for the first 10 inches from the bottom, the second area 6.02 serves for the second 10 inches, viz. from 10 to 20 inches, the third area 6.55 serves for the third 10 inches, viz. from 20 to 30, and the fourth area 6.96, serves from 30 to 38, the last 8 inches of the copper's depth; as for example, at 15.5 inches deep of this copper, how many gallons of ale would it contain?

$$\begin{array}{r} 10 \times 5.46 = 54.6 \\ 5.5 \times 6.02 = 33.1 \\ \hline \end{array}$$

The content at 15.5 inches = 87.7 ale gallons.

When the depth is 10 inches, the trouble of multiplication is saved, for it is only removing the dot in the area one place farther to the right hand, as $5.46 \times 10 = 54.6$, and so of any other. When it is 20 or 30 inches deep, you need only to remove the dots of those areas one place farther to the right hand, and you will have the several contents at those depths; and the content of the odd inches and tenths may be found by the rule; as for example, when the depth is 5.5 inches above 10, set unity on B to the second area on A = 6.02, then against 5.5 on B is 33.1 on A = the content before found. By thus finding the contents of the several parts of the depth, and adding them together, you will gain the whole content at any depth whatsoever.

A R T.

ART. IV.

To gage and fix an under back, whose bases are ovals or ellipses, whether they be equal or unequal, provided they be parallel to each other.

RULE.

1. **W**ITH some convenient instrument take the transverse and conjugate diameters about the middle of the depth, and also the depth, which is commonly about 10 or 12 inches, and set them down on a piece of paper.

2. Find the area on 1 inch by Art. x. of Chap. vii. which, with the other dimensions, transfer to the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

EXAMPLE.

Let the figure in page 74 represent the base of the under back, whose transverse diameter = 72 inches, and conjugate diameter = 50 inches, and suppose I find the depth = 10 inches, it is required to be fixed in the dimension book.

OPERATION.

Transf. diam. A B = 72

Conj. diam. c D = 50

Circ. div. = 359.05) 360000 (10.03 = area in ale gs.

35905

...95000
107715

12715 Too much

The area is not quite 10.03, but is nearer to that than 10.02; and because we make use of but two decimals in the area, we always take the second the nearest to the truth. The area thus found, with the other dimensions, transfer to the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

ART. V.

To gage and fix a back or cooler.

DEFINITION.

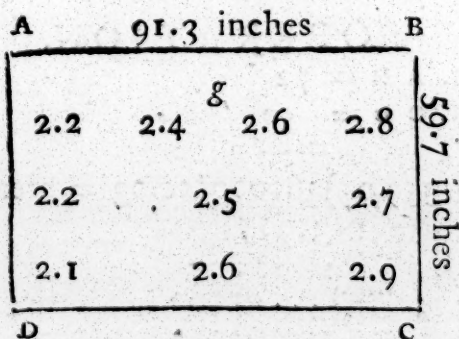
BACKS or coolers are generally made in the form of a square or an oblong, and about 7 or 8 inches deep.

RULE.

1. With a dimension cane, or some other convenient instrument, take the length and breadth of the back about the middle of the depth; which set down on a piece of paper.

2. Find the area at one inch deep by Art. i. and ii. of Chap. vii. which area, with the dimension taken, transfer to the dimension book, according to the specimen exhibited in Chap. viii. and it is done.

A BACK or COOLER.



EXAMPLE.

Let the above figure represent the bottom of the cooler, and suppose I found the length of it A B = 91.3 inches, and the breadth B C = 59.7 inches: to find the area, and fix it in the dimension book.

OPERA.

OPERATION.

Length A B — = 91.3

Breadth B c — = 59.7

6391
8217
4565

Square div. = 282)5450.61(19.33 = area in ale galls.

282

2630

2538

..926

846

801

846

45 Too much.

The area thus found, with the length and breadth, is transferred to the specimen of a dimension book in Art. ix. of Chap. xiii. which was required.

It will not be amiss to try those areas by the rule before you enter them into the dimension book, in order to prevent any mistake that may happen in the operations.

ART. VI.

To find a dipping place in a back or cooler.

BECAUSE backs are not set level, but rather sloping, for the conveniency of the wort's running off; therefore if you should dip in too deep a place, the gage would be too much, and if in too shallow place, the gage would be too little. To prevent this you must

F 5

find

find a proper place to dip always at, that shall neither be too much nor too little.

R U L E.

When there is liquor in the back take as many dips as you think convenient, and add them all together; their sum divide by the number of dips taken, and the quotient will be a mean depth. When this is done, try in several places of the back, till you find a dip that will answer your mean depth; which when found, mark the side of the back right against that place for a constant dipping place.

E X A M P L E.

Suppose I take ten dips in the back A B C D (vide last figure) at the several places where the depths are set in the figure, then if they are added together they will stand as in the margin, and their sum will be = 25, which divided by 10,

or, which is all one, remove the dot one	2.2
place farther to the left hand, gives	2.5
for a mean depth, which by trials I find	2.1
will answer at g; against which I set a	2.6
mark for my constant dipping place, and it	2.8
is done.	2.7
	2.4
	2.6
	2.9
	2.2

10)25.0(2.5

A R T. VII.

To gage and fix a guile tun, or round, in form of a cylinder

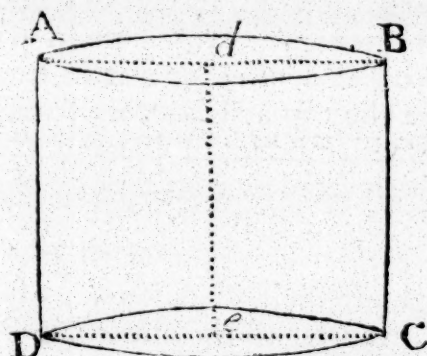
R U L E.

1. **W**ITH some convenient instrument find the diameter of the tun, and also the depth; which set down on a piece of paper.

2. With

2. With that diameter look into the table of ale areas and find the area answering thereto, which area, with the diameter and depth, transfer to the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

A ROUND GUILLE TUN.



EXAMPLE.

Let the figure above represent the tun to be fixed, and suppose I found $AB = DC = 50$ inches, and the depth $de = 40$ inches; to find the area, and fix it in the dimension book.

OPERATION.

By the table of ale areas I find the area answering 50 inches = 6.96, which, with the depth and diameter, I put into the dimension book, according to the specimen exhibited in Chap. xiii. which was required.

Note, all depths which are taken in those utensils that are fixed upon one area, must be multiplied by that area, in order to find the content of the liquor in it at any time. As for example: Suppose I had a gage of ale in this tun of 17.3 inches deep, what is the content of that gage?

Operation by the rule.

on B.	on A.	on B.	on A.	
As 1	: 6.96	: 17.3	: 120.4 = ale gallons.	
Unity.	Area.	Depth.	Content	
		F 6		ART.

ART. VIII.

To gage and fix a guile tun in the form of the frustum of a cone.

R U L E.

1. **W**ITH some convenient instrument take mean diameters between every ten inches, in the same manner as the copper was done in Art. iii. of this chapter, and also the depth; or if it be the frustum of a right cone, take the top and bottom diameters, and by Art. x. of Chap. viii. find mean diameters between every ten inches; which set down on a piece of paper.

2. With those mean diameters seek in the table of ale areas for the respective areas answering each diameter, and set them against their diameters, as below, which, with the depth, transfer to the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

E X A M P L E.

Let the mash tun in Art. i. of this chapter, also represent the guile tun required to be fixed (which is often the case that one tun serves for both mashing and working of beer) whose top diameter $AB = 70$ inches, bottom diameter $DC = 50.4$ inches, and depth $ao = 40$ inches; to find mean diameters between every 10 inches, and to fix it in the dimension book.

O P E R A T I O N.

$$AB = 70.$$

$$DC = 50.4$$

$$40 \overline{) 19.6} (.49 = \text{com. factor.}$$

$$160$$

$$360$$

$$360$$

$$0$$

Factor. Diff. B. Diam. M. Diam. Areas.

$$\begin{array}{l} 5 \times, 49 = 2.4 + 50.4 = 52.8 \text{ } a \text{ } a \left\{ \begin{array}{l} 7.76 \\ 9.27 \\ 10.91 \\ 12.69 \end{array} \right\} \\ 15 \times, 49 = 7.3 + 50.4 = 57.7 \text{ } b \text{ } b \\ 25 \times, 49 = 12.2 + 50.4 = 62.6 \text{ } c \text{ } c \\ 35 \times, 49 = 17.1 + 50.4 = 67.5 \text{ } d \text{ } d \end{array}$$

Thus have I found the mean diameters in the middle between every ten inches, as above, and the areas answering each diameter, as per the tables of ale areas, are ſet againſt them, which, with the depth, I have placed in the ſpecimen of a dimension book exhibited in Chap. xiii. which was required.

ART. IX.

To gage and fix a guile tun whoſe baſes are elliptical or oval, but unequal, called a cylindroid.

RULE.

1. **M**AKE marks with chalk at the extremes both of the tranſverſe and conjugate diameters, according to the directions in Art. iii. of this chapter, in the middle between every 10 inches of the depth, and with the dimension cane or Branan's rule meaſure the tranſverſe and conjugate diameters at thoſe marks, beginning at the lower baſe; take alſo the depth, which dimensions ſet down on a piece of paper, as below.

2. By Art. x. of Chap. vii. find the areas of the ſeveral diameters, and ſet them down, as under, againſt their reſpective diameters, which, with the depth, transfer to the dimension book, according to the ſpecimen exhibited in Chap. xiii. and it is done.

EXAMPLE.

Suppoſe I found the depth of the tun = 30 inches, and the tranſverſe diameter at 5 inches deep = 42.2 inches, and the conjugate diameter = 34.2 inches; at

15 inches deep the transverse diameter = 43.4, and the conjugate = 36.5; and at 25 inches deep the transverse diameter = 44.4, and the conjugate = 38.4 inches; to find the several areas, and to fix the same in the dimension book.

The proportion for the areas by pen or rule is

$$\text{as } \left\{ \begin{array}{l} 359.05 : 44.4 : : 38.4 : 4.75 \\ 359.05 : 43.4 : : 36.5 : 4.41 \\ 359.05 : 42.2 : : 34.2 : 4.02 \end{array} \right\} \text{ at } \left\{ \begin{array}{l} 25 \\ 15 \\ 5 \end{array} \right\} \text{ inches.}$$

Cir. div. Transf. Conj. Areas.

	Diameters.		Areas.	
thus I have found if the	Transf. = 44.4	}	= 4.75	which areas and diame- ters, with the depths, are transferred to the speci- men of a dimension book exhibited in Chap. xiii. which was required.
	Conj. = 38.4			
	Transf. = 43.4	}	= 4.41	
	Conj. = 36.5			
	Transf. = 42.2	}	= 4.02	
	Conj. = 34.2			

ART. X.

To gage and fix a guile tun made of a hogshead, or pipe, standing upon one head, the other being cut off. See the figure following.

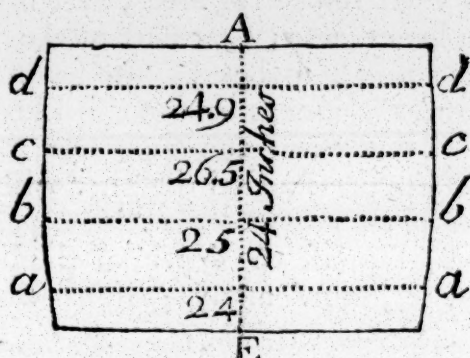
RULE.

1. **FIND** mean diameters in the middle between every 6 or 10 inches, according to the directions in Art. iii. of this chapter, for the copper. If the tun is large, and the diameters differ much, you may find diameters in the middle of every 4 inches; which diameters, and also the depth, set down on a piece of paper.

2. With these diameters find their respective areas in the table of ale areas, and set them against their diameters, as below, which, with the depth, transfer to the dimension book, according to the specimen exhibited in Chap. xiii. and it is done.

A GUILLE

A GUILLE TUN.



EXAMPLE.

Let the above figure represent the tun to be gaged and fixed, and suppose I found the depth of the tun $A E = 24$ inches, and the diameter $a a$, 3 inches from the bottom, $= 24$ inches, the diameter $b b$, 9 inches from the bottom, $= 25$ inches, the diameter $c c$, 15 inches from the bottom, $= 26.5$ inches, and the diameter $d d$, 21 inches from the bottom, $= 24.9$ inches; to find the several areas, and fix it in the dimension book.

OPERATION.

By the table of ale areas I find the areas answering each diameter as follows.

Diameters.		Areas.	
against	$d d = 24.9$	are	1.73
the dia-	$c c = 26.5$	those	1.96
meters	$b b = 25.$	areas	1.74
	$a a = 24.$		1.60

in the table of ale areas

It will not be amiss to try your work by the rule, by setting unity on c to the circular ale gage point on D , then, if there is no mistake made, against the several diameters on D are the areas before found on c , which with the depth, I have transferred to the specimen of a dimension book in Chap. xiii. as was required.

Note,

Note, all depths that are taken in this tun must be multiplied by their respective areas, viz. if 6 inches, or under, by the lower area; that part of the depth which is between 6 and 12 inches by the second area; that between 12 and 18 inches by the third area; and that part between 18 and 24 by the upper area: which several contents of the parts added together will be the content of the whole tun.

For example, suppose the depth of the liquor in this tun was 20 inches, how many gallons would it contain?

OPERATION.

$$6 \times 1.60 = 9.6$$

$$6 \times 1.74 = 10.44$$

$$6 \times 1.96 = 11.76$$

$$2 \times 1.73 = 3.46$$

$$\text{Depth } 20 = 35.26 = \text{content.}$$



ART. XI.

To gage and fix a guile tun, whose parallel bases are both squares or parallelograms, but unequal.

RULE.

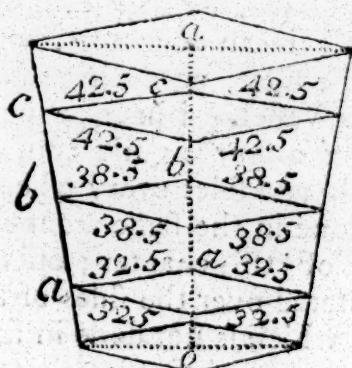
1. WITH some convenient instrument find the depth of the tun, and also the length and breadth in the middle between every 10 inches, by the directions in Art. iii. of this chapter, which set down on a piece of paper.

2. By Art. i. or ii. Chap. vii. find the areas at the several depths, and set them down against their respective dimensions, as in the following work, which, with the depth, transfer to the dimension book, according to the specimen of a dimension book in Chap. xiii. and it is done.

EXAM-

EXAMPLE.

Suppoſe the following figure to repreſent the tun, which is the fruſtum of a ſquare pyramid, whoſe depth $a o$ I found = 30 inches; and the length of the ſide $a a$, 5 inches from the bottom, = 32.5; and at 15 inches from the bottom, the ſide $b b$ = 38.5 inches; and at 25 inches deep, the ſide = 42.5 inches; to find the areas at the ſeveral depths, and to fix the ſame in the dimension book.



OPERATION.

Side $a a$ 32.5 32.5 — 1625 650 Sq. 975 Div. — area 282) 1056.25 (3.74 846 — 2102 1974 — 1285 1128 — Rem. 157	Side $b b$ 38.5 38.5 — 1925 3080 1155 — area 1482.25 (5.25 1410 — .. 722 564 — 1585 1410 — .. 175	Side $c c$ 42.5 42.5 — 2125 850 1700 — area 1806.25 (6.41 1692 — 1142 1128 — .. 145 282 — 137
---	---	---

Thus

Thus have $\left\{ \begin{array}{l} L \ 32.5 \\ B \ 32.5 \end{array} \right\}$ the $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ Which sides
 I found $\left\{ \begin{array}{l} L \ 38.5 \\ B \ 38.5 \end{array} \right\}$ areas will be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ and areas, with
 that if the $\left\{ \begin{array}{l} L \ 42.5 \\ B \ 42.5 \end{array} \right\}$ will be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ the depth, are
 sides are $\left\{ \begin{array}{l} L \ 42.5 \\ B \ 42.5 \end{array} \right\}$ be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ transferred to
 the specimen

of a dimension book exhibited in Chap. xiii. as was required.

In this manner may victuallers utensils of any other form be gaged and fixed, be they ever so irregular, by taking more or less mean diameters, or mean lengths and breadths of the sides, according to the regularity or irregularity of the vessel. I have purposely omitted to give an example of a square guile tun, whose bases are equal and parallel, and sides every where straight lines, because that is done in the same manner as the back or cooler in Art. v. of this chapter; only the depth must be taken, and entered over the sides of the square in the dimension book, which is not done in fixing of the back or cooler.

ART. XII.

To gage all sorts of by-tubs

DEFINITION.

BY-tubs are such as are not fixed upon areas in the dimension book, but required other dimensions to be taken besides the depth, in order to find the content, viz. the length and breadth, or mean diameter.

RULE.

Take the depth of the liquor in the vessel, and also a mean diameter, by laying the gaging rod over the surface of the liquor. If the top and bottom diameters are unequal, you must guess at a mean diameter; or if the

the tub be elliptical, lay the gaging rod across between the transverse and conjugate diameters, and guess at a mean diameter that will reduce the same to a cylinder, which mean diameter set down in your book over the depth, with the character or quality of the liquor contained in the tub over it.

2. To gage by-tubs that have squares or parallelograms for their bases, whether they be equal or unequal.

RULE.

Take the mean length and breadth of the tub, and also the depth of the liquor in it, which set down in your book, viz. the depth under, next the breadth with the letter B. against it for breadth, and above that the length with the letter L. against it for length, and lastly set the character over it, and it is done.

In some places it is customary to cool the worts in brass pans or kettles. In such vessels the diameter is found as above; but the depth must not be taken in the middle, but about four or five inches from the side, according to the size of it, because these kind of vessels have a fall in the middle.

Sometimes worts are cooled in bowls. In such cases take the diameter of the liquor's surface, and also the depth in the middle of the bowl; but set down only half that depth in your book, which will answer pretty near the truth in practice; but experience will be the best guide in such cases

ART. XIII.

To deduct the heat out of warm worts in casting up the gages.

IT has been found by experience that 10 gallons of hot wort will be but nine gallons when cold, therefore, by act of parliament, there should be an allowance made of one gallon in every ten gallons gaged hot. To do this observe the following directions.

RULE.

R U L E.

This deduction may be made either by subtraction or multiplication.

1. By subtraction. Set down the number of warm gallons twice; but the under number must be set one figure farther to the right hand, or, which is the same, remove the dot of the subtrahend one place farther to the left hand, and it is divided by 10, and this tenth part subtracted from the number of warm gallons will leave the quantity of wort to be charged.

2. By multiplication. Multiply the number of warm gallons by .9 a decimal, and the product will be neat gallons.

E X A M P L E.

Suppose a gage of warm wort were 45 gallons, what must be charged?

O P E R A T I O N.

By SUBTRACTION.

$$\begin{array}{r} 45 \\ 4.5 \\ \hline \end{array}$$

40.5 = neat gallons.

By MULTIPLICATION.

$$\begin{array}{r} 45 \\ .9 \\ \hline \end{array}$$

40.5 = neat gallons.

Thus you see that by either method the answer is the same; but this may be performed by the rule without knowing the number of warm gallons at all; but to do this there must first be a divisor found.

And as divisors are the reciprocals of multipliers, it is only dividing unity by .9, and the quotient will be a proper divisor to effect the same. For example:

.9)1.0(1.11 = Divisor.

$$\begin{array}{r} 9 \\ \hline .10 \\ 9 \\ \hline .10 \\ .9 \\ \hline \end{array}$$

Remains 1

Then

Then the proportion will be for the example before,

	on B.	on A.	on B.	on A.
As 1.11	:	1	:	45
Divisor.	Unity.	Warm	Neat	
		galls.	galls.	

But when the number of warm gallons are not known, and you have the depth of the liquor in a fixed utensil, and also the area of that depth given, then the proportion is

As the divisor is to the area, so is the depth to the content in neat gallons. As for example, suppose the depth of the liquor was 9 inches, and the area belonging to that depth was 5 gallons, how many neat gallons would it contain?

OPERATION.

	on B.	on A.	on B.	on A.
As 1.11	:	5	:	9
Divisor.	Area.	Depth.	Content.	

For $9 \times 5 = 45$, and one tenth of 45 being deducted from it leaves 40.5, as above; and so of any other. Therefore it will be necessary to make a mark at 1.11, either upon the line A or B, for the conveniency of casting up of warm gages.

If the warm worts are in circular by-tubs, and the depth under diameters, then, instead of the gage point 18.95 on the line D, make use of 20, to which set the depth on C, and against any diameter on D is the content in neat gallons on C. As for example, suppose the diameter of a by-tub = 30 inches, and the depth of warm wort in it = 12 inches, how many neat gallons does it contain?

OPERATION.

	on D.	on C.	on D.	on C.
As 20	:	12	:	30
Warm gage	Depth.	Diam.	Content.	
point				

The

The heat is deducted out of common brewers warm gages in a different manner, by reason of the different denominations in the given number of barrels, &c. Therefore to do this observe the following directions: for every 10 barrels set down 1 under the barrels, and what remains above 10 barrels reduce into firkins, and add the firkins (if any) to that sum; and for every 10 firkins set down 1 under the firkins, and what remains above 10 firkins reduce into gallons, and add their sum to the gallons; and for every 10 gallons set down 1 under the gallons: then subtract the under number thus found from the given number of barrels, &c. above it, and the remainder will be equal to the neat gallons. For example:

	B.	F.	G.	
Let the warm barrels, &c. =	16	3	6.	To find
the neat barrels	1	2	6.5	
	15	0	8	= neat barrels.

In the above example I set down 1 for the 10 barrels under barrels, and reduce the 6 barrels into firkins, = 24, to which I add 3 = 2.7F. then for the 20 firkins I set down 2 under 20 firkins, and reduce the other 7 into gallons = 59.5, to which I add 6, which = 65.5, for which I set 6.5 under the gallons, and subtract the lower number from that above, and it is done.

EXAMPLE II.

	B.	F.	G.	
Let the warm barrels, &c. =	86	0	4	To find
the neat barrels, &c.	8	2	4	
	77	2	0	= neat barrels.

EXAM-

EXAMPLE III.

B. F. G.

Let the warm barrels, &c. = 9 : 3 : 0. To find
the neat gallons

0 : 3 : 7.5

8 : 3 : 1 = neat
barrels.

And in this manner may the heat be deducted out of
any number of barrels, &c. whatsoever.

ART. XIV.

To find new gage points.

WHEREAS it may happen sometimes in business,
that when one of the given numbers is set to the
gage point the other given number will fall off the
rule. To remedy this there must be new gage points
found.

Thus,

Set unity on c to the old gage point on d, and
against the other 1 on c is the new gage point on d.

EXAMPLE I.

Suppose it were required to find a new gage point to
18.95 the old ale gage point.

OPERATION.

Set unity on c to 18.95 on d, and against the other
1 on c is 59.92, the new gage point on d for circular
ale gallons.

EXAMPLE II.

To find a new gage point for wine measure.

OPERATION.

Set unity on c to 17.15 on d, and against the other
1 on c is 54.22 on d, the new gage point for circular
wine gallons.

In

In like manner may any other new gage point be found to any of the old gage points in the table p. 57.

These second gage points are the square roots of the divisors multiplied by 10. As for instance, the circular divisor for ale gallons = 359.05, which multiplied by 10 = 3590.5, whose square root = 59.92, the new gage point for ale gallons, as before found; and 294.12 the circular divisor for wine gallons, multiplied by 10 = 2941.2, whose square root = 54.22, the new gage point for wine gallons, as before found; and so for any other new gage point.

The use of the new gage points

Suppose the depth of a vessel be = 8 in. ¹⁰/₃, and the diameter = 9 inches, what is the content in ale gallons?

OPERATION.

on D.	on C.	on D.	on C.
As 59.92	: 8	: : 9	: 1.8 = ale gallons.
New ale	Depth.	Diam.	Content.
gage pt.			

But if the content were required in wine gallons,

on D.	on C.	on D.	on C.
As 54.22	: 8	: : 9	: 2.20 = wine gallons.
New wine	Depth.	Diam.	Content.
gage pt.			

XX

CHAP. X.

THIS chapter treats of gaging and inching, &c. of all sorts of common brewers open utensils according to the common methods made use of by the officers of the excise; and from thence is deduced a little table-book, which is placed at the latter end in the same form, as if it was a real table-book, which may serve as a precedent for young officers to apply to whenever they may have occasion; for it

it may sometimes happen that a brewhouse may be erected in a division where there was none before.

I shall begin with the mash tun, and proceed in the order they must stand in the table book.

ART. I.

To gage and inch a mash tun whose parallel bases are equal squares or parallelograms, and its sides every where straight lines.

RULE.

1. **W**ITH some convenient instrument take the length and breadth, and also the depth of the tun.

2. By Art. ii. Chap. ix. find the area upon one inch deep, and multiply that area by the depth, and the product will be the content of the tun in gallons.

3. Reduce both the area and the whole content of the tun into quarters, bushels, and gallons.

4. To find the content upon every inch of the tun's depth, add the area of the first inch, reduced, to itself, and the sum will be the content at 2 inches deep; to this sum add the area of the first inch, and you will have the content at 3 inches deep; and in this manner keep continually adding the area of 1 inch to the content of the 3d, 4th, 5th, 6th, &c. inches, till you arrive at the depth of the whole tun; which last will be equal to the content of the whole tun if the work be done right, if otherwise not.

EXAMPLE.

Suppose I found the depth of a square mash tun = 30 inches, the length of the bases = 119 inches, and the breadth = 102 inches: to find the content of the whole tun, and also the content at every inch of its depth.

OPERATION.

Length of bafes = 119

Breadth of bafes = 102

 238
 1190

 Sq. div. = 227)12138(
 1135

 53.47 = Area at 1 inch
 30 = depth.

 ..788
 8)53.47 = area in 681
 galls. —

8)1604.10 = cont. in galls.

 6:5.47 = area in 1070
 bushels, &c. 908

8)200 4.1 = cont. in bushels.

q. b. g.

25 : 0 : 4.1 = content in
 quarters, &c.

 1620
 1589

Remains — .. 31

Thus I have found the content of the whole tun =
 25 qrs. 0 bs. 4.1 gs. and the area at 1 inch deep =
 0 qrs. 6 bs. 5.47gs. The next thing to be done is to
 find the content at every intermediate inch of the depth,
 which is done thus:

Q. B. G.

0 : 6 : 5.47 = Content at 1 inch.

0 : 6 : 5.47

 1 : 5 : 2.94 = Content at 2 inches.

0 : 6 : 5.47

 2 : 4 : 0.41 = Content at 3 inches.

0 : 6 : 5.47

 3 : 2 : 5.88 = Content at 4 inches.

0 : 6 : 5.47

 4 : 1 : 3.35 = Content at 5 inches.

And

And in this manner I proceed till I come to 30 inches, the whole depth of the tun, which I find to be 25 qs. 0 bs. 4.1 gs. = the content before found, which proves the work to be done right. The several contents thus found are placed in the table book, page 140, against their respective inches throughout the whole depth. The dimensions and area at one inch, with the content, are placed in the front of the table book, in page 139, which was required.

In filling up of the table I have rejected the odd tenths if they were under 5, and when they exceeded 5, I have added one more to the even gallons.

If the tun to be inched had been a cylinder, the diameter must have been squared, and that square divided by the circular divisor for mash tuns in page 57, and the rest of the work would be just the same as for the square mash tun; or if it had been the frustum of a cone, then I should find mean diameters in the middle between every 6 or 10 inches, and then have found the areas of those diameters, which areas added for every respective inch to which they belong, will give the content at every inch of the tun's depth, as will appear more clearly by the following example of the copper.

ART. II.

To gage and inch a copper with a rising crown, and make an allowance for the same. See the following figure.

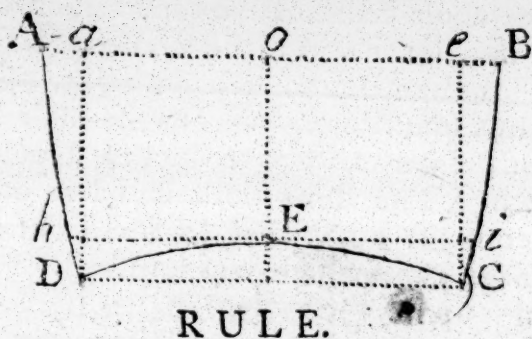
EXAMPLE.

LET the figure A B C D represent the copper to be gaged and inched.

G 2

A COP.

A COPPER.



1. Take a small cord or thread, and fasten one end of it at A, and extend the other end to the opposite side of the copper at B, where make it fast; then with some convenient instrument find the greatest depth of it, which is $= aD = eC$; that is the nearest distance from the thread to the bottom at D, and suppose it = 30 inches.

In like manner take the least depth of the copper = $\bullet E$, which is the nearest distance from the thread to the top of the crown at E, which suppose to be = 27 inches, then $30 - 27 = 3$ inches, the height of the crown.

2. To find the diameter at the bottom of the crown, measure A B the top diameter, which suppose = 90 inches; then hold a thread with a plummet at the end of it, so as that the plummet may hang over D or C, then measure the distance between the edge of the copper and the place where the threads cut each other = $Aa = eB$, which suppose = 5 inches, and $5 + 5 = 10$ subtracted from the top diameter = 90 leaves $80 = DC$, the diameter at the bottom of the crown.

3. To find the content of the liquor required to cover the crown, measure the diameter hi , which touches the top of the crown, and suppose it = 82 inches; then by having the top and bottom diameters, and the altitude of the frustum of a cone, or that part $hicD$, the content of that part may be found by Art. iv. Chap. viii. from which the content of the crown must be subtracted.

4. The content of the crown may be found by Art. viii.

viii. and ix. of Chap. viii. as the frustum of a sphere; or thus, multiply the area of the bottom diameter = $d c$ by half the altitude of the crown, and the product will be nearly equal to the content of the crown, which subtracted from the part $b i c d$ will leave the quantity of liquor required to cover the crown.

Operation for finding the quantity of liquor to cover the crown.

Diam. $b i$	= 82	} Areas.	18.727	Area of the diam. $d c$ = 17.825 $\frac{1}{2}$ alt. = 1.5
Diam. $d c$	= 80		17.825	
Mean diam.	= 80.9		<u>18.23</u>	
The content of the part $b i c d$ — — }			= 54.782	89125
Content of the crown			= <u>26.7375</u>	<u>17825</u>
Content of liquor to cover the crown }			= 28.0445	26.7375

But the content of the liquor to cover the crown may be found otherwise, viz. From the area of the diameter at the top of the crown = $b i$ subtract $1\frac{1}{2}$ of the area of the crown's height, and the remainder multiply by half the height of the crown, whose product will be equal to the number of gallons to cover the crown; or it may be found by measure (which is the most practical method) without finding the dimensions of the crown at all, by pouring in water from a gallon, or some other known measure, till the crown is covered.

At the same time take mean diameters between every 6 or 10 inches, beginning from the top of the copper towards the bottom: as suppose in this example I found the first mean diameter at 5 inches from the top = 88 inches, at 15 inches from the top = 85.5 inches, and at 25 inches from the top = 82.5 inches, as they are set down in the second column of the following table.

By the table of ale areas I find their respective areas as they are set against these diameters in the third column; and the contents of the several parts of the

depth in gallons are placed in the fourth column ; which several contents are reduced to barrels, &c. and placed in the three last columns.

Pts. of Depth.	Diam.	Area.	Content in Gallons.	Content in		
				B.	F.	G.
10	88.0	21.568	215.680	6	1	3.180
10	85.5	20.360	203.600	5	3	8.100
07	82.5	18.956	132.692	3	3	5.192
03	To cover the crown.		28.044	0	3	2.544
30	Content of copper.		580.016	17	0	2.016

The two upper contents in gallons are found by removing the dot in the areas one place farther to the right hand ; the last content is found by multiplying its correspondent area by the depth 7.

The next thing is to find the content upon every inch of the copper's depth.

R U L E.

From the whole content of the copper reduced into barrels, &c. subtract the area of the first 10 inches, and the remainder will be the content when one inch is dry ; and so continue subtracting that area from the remainder until 10 inches are dry, and you will gain the contents of the first 10 dry inches. See the work.

OPERATION.

Whole	Continued.	Continued.
cont. B. F. G.	10 = 10:2:7.336	4:2:7.736
= 17:0:2.016	Note 0:2:3.360	Note 0:2:1.956
Sub. 0:2:4.568		
1 = 16:1:5.948	11 = 10:0:3.976	21 = 4:0:5.780
0:2:4.568	0:2:3.360	0:2:1.956
2 = 15:3:1.380	12 = 9:2:0.616	22 = 3:2:3.824
0:2:4.568	0:2:3.360	0:2:1.956
3 = 15:0:5.312	13 = 8:3:5.756	23 = 3:0:1.868
0:2:4.568	0:2:3.360	0:2:1.956
4 = 14:2:0.744	14 = 8:1:2.396	24 = 2:1:8.412
0:2:4.568	0:2:3.360	0:2:1.956
5 = 13:3:4.676	15 = 7:2:7.536	25 = 1:3:6.456
0:2:4.568	0:2:3.360	0:2:1.956
6 = 13:1:0.108	16 = 7:0:4.176	26 = 1:1:4.500
0:2:4.568	0:2:3.360	0:2:1.956
7 = 12:2:4.040	17 = 6:2:0.816	27 = 0:3:2.544
0:2:4.568	0:2:3.360	Crown 0:3:2.544
8 = 11:3:7.972	18 = 5:3:5.956	Rem.
0:2:4.568	0:2:3.360	= 0:0:0.000
9 = 11:1:3.404	19 = 5:1:2.596	
0:2:4.568	0:2:3.360	
10 = 10:2:7.336	20 = 4:2:7.736	
6:1:3.180	6:1:3.180	
	5:3:8.100	
17:0:2.016	17:0:2.016	
	G 4	

When

When I have finished the subtraction with the first area, which is at the 10th dry inch, I add the content of the first 10 inches to the remainder, in order to prove my work, whose sum, if right, will be equal to the content of the copper, as before found.

Then I take the second area and subtract it from the last remainder, and so continue to do till I come to the 20th dry inch and then to prove my work I add the contents of the first and second 10 inches to the last remainder, whose sum, if right, will be also equal to the whole content of the copper.

Lastly, I take the third area and subtract it from the last remainder, and so proceed till I come to the 27th dry inch, whose remainder, if the work be done right, will equal the quantity of liquor to cover the crown, as before found. Thus having finished the subtraction, I transfer the several contents to the nearest even gallon to the table book in page 140, and it is done.

I have been the longer on this article in order to render the affair of inching, &c. not only of the copper, but also all other sorts of utensils that stand upon several areas, as plain as possible; for the content at every inch of the depth is found after the same manner.

In adding and subtracting of barrels, firkins, and gallons, there will arise some difficulty by reason of the odd half gallons, or 5 tenths, that are contained in a firkin of ale; which I shall endeavour to remove by the next article, and to render the business of addition and subtraction of those kind of numbers as easy as if the firkins consisted only of whole gallons.

ART. III.

To add and subtract barrels, firkins, and gallons, &c.

I. Of Addition.

RULE.

IF the two giving numbers of gallons and tenths to be added together be under a firkin, add them as in common arithmetic: but if they are above a firkin (which will

will appear by inspection) add the thirds and seconds together (if there be any) as in common; and when you come to the primes, always add 5 tenths to the sum of the primes, and set down the excess above 10 underneath the primes, and for 10 primes carry 1 to the gallons; then as many gallons as there are above 9 set down under the gallons, and for the 9 gallons carry 1 to the firkins, and then proceed as usual.

EXAMPLE.

$$\begin{array}{r} \text{To} \quad \text{---} \quad 7.854 \\ \text{Add} \quad \text{---} \quad 3.645 \\ \hline \end{array}$$

1:2.999

Here by inspection I see that the sum to be added will exceed a firkin; therefore beginning at the thirds place, I say, $5 + 4 = 9$, which I set underneath; then for the seconds I say $4 + 5 = 9$, which I set under the seconds; then for the primes I say $6 + 8 = 14$, and 5 more I borrow $= 19$. I set down the 9 under the primes, and carry 1 to the gallons; and $1 + 3 = 4$, and $7 = 11$, which is 2 above 9, and which I set down under the gallons, and for the 9 gallons I carry 1 to the firkins; which is 1 F. 2 Gs. and ,999 thousandth parts of a gallon. This is so easy, that it requires no more examples.

II. *Of Subtraction.*

RULE.

1. If the minuend be greater than the subtrahend, then subtract as usual. But if the minuend be less than the subtrahend (which will appear by inspection) subtract the thirds and seconds as in common arithmetic: but when you come to the primes add 5 tenths to the primes of your minuend, and subtract the figure below from the augmented prime, and set down the remainder underneath the primes; then add 8 to the gallons of the minuend, and subtract the gallons from

G 5

that

Add 1.5 (the complement of 8.5 to 10 gallons) to the common area that is to be added or subtracted, and the area thus increased made use of in addition, instead of the common area, in case the sum of the fractions to be added exceed a firkin; and in subtraction, where the subtrahend exceeds the minuend. To make this more intelligible, I shall subjoin the two following examples, one in addition and the other in subtraction, to three inches of the depth. The operation will be the same for the whole depth, provided the areas are alike from top to bottom; but whenever the areas vary, the increase of 1.5 must be made to the new area.

$$\begin{array}{r} \text{B. F. G.} \\ \text{Let the com. area to be added} = 0 \ 0 \ 4.855 \\ \text{add} \quad 0 \ 0 \ 1.5 \\ \hline \end{array}$$

$$\text{Increased areas} \quad 0 \ 0 \ 6.355$$

$$\begin{array}{r} \text{B. F. G.} \\ \text{And to be subtracted} = 0 \ 2 \ 4.568 \\ \quad \quad \quad 0 \ 0 \ 1.5 \\ \hline \end{array}$$

$$\text{Increased areas} \quad 0 \ 2 \ 6.068$$

ADDITION.

					B. F. G.
Area	—	—	—	—	= 0 0 4.855
Increased area	—	—	—	—	= 0 0 6.355
1 Inch	—	—	—	—	= 0 1 1.210
Common area	—	—	—	—	= 0 0 4.855
2 Inch	—	—	—	—	= 0 1 6.065
Increased area	—	—	—	—	= 0 0 6.355
3 Inch	—	—	—	—	= 0 2 2.420

SUBTRACTION.

					B.	F.	G.
Whole content	—	—	—	=	17	0	2.016
Increased area	—	—	—	=	0	2	6.068
1 Inch	—	—	—	=	16	1	5.948
Common area	—	—	—	=	0	2	4.568
2 Inch	—	—	—	=	15	3	1.380
Increased area	—	—	—	=	0	2	6.068
3 Inch	—	—	—	=	15	0	5.312

And in this manner may any number of gallons and firkins be added and subtracted with as much ease and expedition as if they were whole numbers.

ART. IV.

To gage and inch an under-back whose bases are equal squares, or parallelograms, and its sides perpendicular to the bases.

RULE.

1. **WITH** some convenient instrument take the length of the back, which suppose here to be = 148 inches, and also the breadth = 112 inches, and suppose the depth = 10 inches.

2. By Art. i. for a square, or Art. ii. Chap. vii. for a parallelogram, find the area upon 1 inch, which multiply by the depth, and the product will be equal to the content of the whole back.

3. To find the content at every inch of the tun's depth, reduce the area of 1 inch into barrels, firkins, and gallons, which will be equal to the content of the first inch; and that sum added to itself will be the content of the second inch; and to the content of 2 inches add the content of the first inch, and you will have the content at three inches deep; and thus, by continually adding the content of the first inch to the contents

of

of 3, 4, 5, 6, &c. inches till the whole depth is completed, you will have the content of the back at every inch of its depth. See the work.

OPERATION.

$$\begin{array}{r}
 148 \\
 112 \\
 \hline
 296 \\
 148 \\
 148 \\
 \hline
 282)16576(\quad 58.78 = \text{area in gallons.} \\
 1410 \quad \quad 10 = \text{depth.} \\
 \hline
 .2476 \quad 587.80 = \text{content in gallons.} \\
 2256 \\
 \hline
 \end{array}$$

$$.2200$$

$$1974$$

$$.2260$$

$$2256$$

B. F. G.

$$17 : 1 : 1.3 = \text{cont. reduced to barrels, \&c.}$$

Rem. — ...4

The area of 1 inch = 58.78 reduced to barrels, fir-

kins, and gallons, is = $\begin{array}{r} \text{B. F. G.} \\ 1 : 2 : 7.78 \\ 1 : 2 : 7.78 \end{array}$

Content at 2 inches = $\begin{array}{r} 3 : 1 : 7.06 \\ 1 : 2 : 7.78 \end{array}$

Content at 3 inches = $\begin{array}{r} 5 : 0 : 6.34 \\ 1 : 2 : 7.78 \end{array}$

Content at 4 inches = $6 : 3 : 5.62$

And

And thus, by continuing the addition to 10 inches = the depth of the back, I found the contents at every inch of the same as they are placed in the table, page 140, which is so easy that it is quite needless to insert the whole process. Only here observe, that if the content of the last inch is equal to the content of the back before found, the work is done right, otherwise not.

ART. V.

To gage a back or cooler, and to find the content at every tenth of an inch, commonly called tenthing a back.

R U L E.

1. **W**ITH some convenient instrument take the length and breadth of the back.

2. By Art. ii. Chap. vii. I find the area or content on one inch deep, which reduce to barrels, firkins, and gallons; find the one tenth of that area by removing the dot one place farther to the left hand, which also reduce to barrels, &c.

3. To find the content at every tenth, add the area or content of the first tenth to itself, and you will have the content at 2 tenths; again, add the content of the first tenth to the content of the second tenth, and you will have the content of three tenths; and thus, by continually adding the content of the first tenth for every tenth, you may tenth a back to what depth you please. But if you continue the table to 5 or 6 inches of the back's depth it will be sufficient in most cases, because the worts are seldom put deeper in them than 6 inches for cooling; and if the gage taken should exceed the limits of the table, it is only taking the depth at twice out of the tables, and adding the content of each depth together, and you will have the content at the whole depth of the gage.

E X A M P L E.

Suppose I found the length of the back = 272 inches, of the breadth = 90.8 inches: to find the content at every tenth of the first 3 inches of the back's depth.

O P E.

O P E R A T I O N.

$$\text{Length} = 272$$

$$\text{Breadth} = 90.8$$

$$\underline{2176}$$

$$\underline{24480}$$

$$282 \overline{)24697.6} (87.58 = \text{area on 1 inch in gallons.}$$

$$\underline{2256}$$

$$\cdot 2137 \quad \text{B. F. G.}$$

$$\underline{1974} \quad 2 : 2 : 2.58 = \text{area 1 inch reduced.}$$

$$\cdot 1636$$

$$\underline{1410}$$

$$\cdot 2260$$

$$\underline{2256}$$

$$\text{Rem.} \quad - \quad \dots 4$$

The area or content of 1 inch being 87.58 gallons, the tenth part of that sum is 8.758 gallons, which reduced is 0 B. 1 F. 0.258 G. and so much will the back hold on every tenth of an inch of its depth. The next thing to be done is to find the content at every tenth of an inch, till you come to the depth required to be inserted in the table, thus.

O P E R A T I O N.

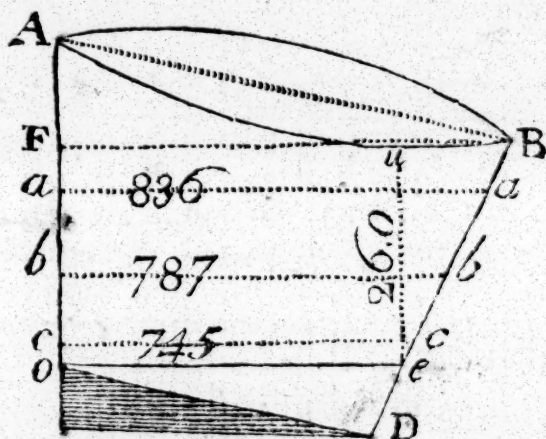
	B. F. G.		B. F. G.
1 Tenth =	0 : 1 : 0.258	6 Tenth =	1 : 2 : 1.548
	<u>0 : 1 : 0.258</u>		<u>0 : 1 : 0.258</u>
2 Tenth =	0 : 2 : 0.516	7 Tenth =	1 : 3 : 1.806
	<u>0 : 1 : 0.258</u>		<u>0 : 1 : 0.258</u>
3 Tenth =	0 : 3 : 0.774	8 Tenth =	2 : 0 : 2.064
	<u>0 : 1 : 0.258</u>		<u>0 : 1 : 0.258</u>
4 Tenth =	1 : 0 : 1.032	9 Tenth =	2 : 1 : 2.322
	<u>0 : 1 : 0.258</u>		<u>0 : 1 : 0.258</u>
5 Tenth =	1 : 1 : 1.290	Con. at }	
	<u>0 : 1 : 0.258</u>	1 inch }	= 2 : 2 : 2.580
6 Tenth =	1 : 2 : 1.548		

Thus

Thus when I come to 1 inch of the back's depth I find the content to be equal to that before found, which proves the work to be true; and in this manner I proceeded to make the table in page 141 to 3 inches of the back's depth, where all the above contents are placed to the nearest even gallon.

ART. VI.

To gage and inch a guile tun in form of the frustum of a cone, and to make an allowance for the drip or fall. See the figure following.



RULE.

1. **C**AUSE water to be poured into the tun till the bottom is just covered, which will be when the water touches the point o, and suppose the measure of the water in this case to be 30.92 gallons.

2. With a quadrant, or a level, find the horizontal line F B thus: fasten a string on the top edge of the tun, as at B, and let an assistant hold it out on the other side. Then rest your level on the same place, and hold it in such a manner as that the bottom of it may be horizontal, which you will find by the plummet. Then let your assistant draw out the string parallel

parallel to the bottom of the level, till it touches the opposite side at *F*, where fasten it; then will *F B* represent the surface of the liquor when the tun in that position is full, and it will be also parallel to the line *o e* on the surface of the liquor which covers the drip.

3. Take the perpendicular depth of the tun, that is, the shortest distance from the water at *e* to the string at *u*, which suppose here to be = 26 inches = *o F* = *u e*.

4. Take mean diameters parallel to *F B* and *o e* in the middle between every 6 or 10 inches of that depth; and suppose the first mean diameter *a a* at 5 inches from *F B* = 83.6 inches; the second mean diameter *b b* at 15 inches from *F B* = 78.7 inches; and the third mean diameter *c c* at 23 inches from *F B* = 74.5 inches.

5. By the table of ale areas in Chap. xv. find the areas, and set them against their respective diameters, as in the third column of the following table. Then find the contents of the several depths in gallons, and set them in the fourth column. Lastly, convert these contents into barrels, &c. and place them as in the three last columns of the table.

Depth.	Diam.	Area.	Cont. in galls.	Content in		
				B.	F.	G.
10	83.6	19.465	194.65	5	2	7.65
10	78.7	17.250	172.50	5	0	2.50
06	74.5	15.458	92.75	2	2	7.75
To cover the drip			30.92	0	3	5.42
26	whole content.		490.82	14	1	6.32

Now to find the content at every inch of the depth reduce the first area 19.465 into barrels, &c. and it will be 0 B. 2 F. 2.465 G. which subtracted from the whole content = 14 B. 1 F. 6.32 G. leaves 13 B. 3 F. 3.855 G. = the content when 1 inch is dry. From that remainder subtract the said area, and you will have the content when 2 inches are dry; and thus continue subtracting the first area from the several remainders

mainders until 10 inches are dry, when there will remain 8 B. 2 F. 7.17 Gs. Then take the second area 17.25 converted into firkins, &c. and substract it from the last remainder, and thus continue to do till you have 20 inches dry, when there will remain 3 B. 2 F. 4.67 Gs.

Lastly, take the third area 15.458 converted into barrels, &c. and substract it from the remainder when 20 inches are dry; and thus proceed till you have 26 inches dry, when you will have remain, if no mistake be made, 0 B. 3 F. 5.42 Gs. = the quantity of liquor to cover the bottom as before found.

I have omitted to shew the operation at large, because it is performed just in the same manner as the copper in Art. ii. of this chapter. But the contents at every dry inch in barrels are inserted in the table, page 141, to the nearest even gallon.

N. B. You must make a mark at F for a constant dipping place while the tun is in this position.

If this tun had been a cylinder, or a square, whose bases are equal and parallel, and whose sides are perpendicular to their bases, then it is plain that the dry hoof F A B would be equal to the wet hoof *o e D*, and the solidity of the hoof *o e D* might easily be found either by measure or calculation: for if half the depth of the water at *e D* be multiplied by the area of the base, the product will be the content of the drip, which substracted from the whole content of the tun will leave the content of it in that position, and the dipping place will be the same distance from A as *e* is from D, that is, A F must be = *e D*. In like manner may tuns of any other form be gaged and inched, provided the areas are found at the several depths according to the directions delivered in Chap. vii.

Now having fully explained the nature of gaging and inching, &c. of common brewers utensils, I shall, in the next place, give a precedent of a table-book, which was collected from the foregoing operations.

A Table

A Table Book.

A—— B——, common brewer.

No. and names.		Depth.		Diameter.		Length.		Breadth.		Area.		Content in gallons.		Content in B. F. G.		Content in Q. B. G.	
MT	Copper.	UB	Back	Round.	07	10	10	30.0	119	102	53.47	534.7	-	-	25	0	4.1
		10	—	10	82.5	88.0	85.5	30.0	—	—	21.568	215.680	6	13.180			
		30	—	10	—	—	—	07	—	—	20.360	203.600	5	8.100			
		03	—	06	—	—	—	10	—	—	18.956	132.692	3	5.192			
					To cover the crown.							28.044	0	32.544			
					The whole content.							580.016	17	0.2016			
		10	—	10	148	112	58.78				587.80	17	1	1.3			
		—	—	—	272	90.8	87.58				—	—	2	2.58			
		10	83.6	10	—	—	19.465				194.65	5	2	7.65			
		10	78.7	10	—	—	17.250				172.50	5	0	2.50			
		06	74.5	06	—	—	15.458				92.75	2	2	7.75			
					To cover the bottom.							30.92	0	3	5.42		
					The whole content.							490.82	14	1	6.32		

and 2^d square; and then their dimensions, &c. against them. In this and the following pages are placed the contents at every inch, &c. of the depth of each utensil; their use is for casting up common brewers gages, or finding their content at any inch of the depth. Here it will not be amiss to observe that the wet inches are taken in the mash tun, under back, and backs; but the dry inches in coppers, rounds, and squares.

A Common Brewer's Table Book.

Mash Tun.				Copper.				Under Back.			
Wet inches.	Q.	B.	G.	Dry inch.	B.	F.	G.	Wet inch.	B.	F.	G.
1	0	6	5	Full.	17	0	2	1	1	2	8
2	1	5	3					2	3	1	7
3	2	4	0	1	16	1	6	3	5	0	6
4	3	2	6	2	15	3	1	4	6	3	6
5	4	1	3	3	15	0	5	5	8	2	5
6	5	0	1	4	14	2	1	6	10	1	4
7	5	6	6	5	13	3	5	7	12	0	3
8	6	5	4	6	13	1	0	8	13	3	3
9	7	4	1	7	12	2	4	9	15	2	2
10	8	2	7	8	11	3	8	10	17	1	1
				9	11	1	3	Half			
11	9	1	4	10	10	2	7	inch.	0	3	4
12	10	0	2								
13	10	6	7	11	10	0	4				
14	11	5	5	12	9	2	1				
15	12	4	2	13	8	3	6				
16	13	3	0	14	8	1	2				
17	14	1	5	15	7	2	8				
18	15	0	3	16	7	0	4				
19	15	7	0	17	6	2	1				
20	16	5	6	18	5	3	6				
				19	5	1	3				
21	17	4	3	20	4	2	8				
22	18	3	1	21	4	0	6				
23	19	1	6	22	3	2	4				
24	20	0	3	23	3	0	2				
25	20	7	1	24	2	1	8				
26	21	5	6	25	1	3	6				
27	22	4	4	26	1	1	4				
28	23	3	1	27	0	3	3				
29	24	1	7								
30	25	0	4								
				To cover the Cr.	0	3	3				
				1 half In.	0	1	2.5				
				2 half In.	0	1	2				
				3 half In.	0	1	1.5				

Of the use of the Tables.

EXAMPLE. I.

Suppose the depth of the goods or grains were 18 inches in the mash tun, how many quarters, &c. would it contain at that depth?

I seek for 18 in the first column under wet inches, and against it is 15 Qrs. 0 Bs. 3 Gs. = the content.

Note, In gaging you must take only even inches in the mash tun. But the copper, under-back, rounds, and squares, must be taken to the nearest half inch, and the backs to the nearest tenth.

A Common Brewer's Table Book.

Back.				Round.			
Wet inches and tenths.	B.	F.	G.	Dry inch.	B.	F.	G.
.1	0	1	0	Full.	14	1	6
.2	0	2	1	1	13	3	4
.3	0	3	1	2	13	1	1
.4	1	0	1	3	12	2	7
.5	1	1	1	4	12	0	5
.6	1	2	2	5	11	2	2
.7	1	3	2	6	11	0	0
.8	2	0	2	7	10	1	6
.9	2	1	2	8	9	3	4
1.0	2	2	3	9	9	1	1
.1	2	3	3	10	8	2	7
.2	3	0	3	11	8	0	7
.3	3	1	3	12	7	2	7
.4	3	2	4	13	7	0	6
.5	3	3	4	14	6	2	6
.6	4	0	4	15	6	0	6
.7	4	1	4	16	5	2	6
.8	4	2	5	17	5	0	5
.9	4	3	5	18	4	2	5
2.0	5	0	5	19	4	0	5
.1	5	1	5	20	3	2	5
.2	5	2	6	21	3	0	6
.3	5	3	6	22	2	2	8
.4	6	0	6	23	2	1	1
.5	6	1	6	24	1	3	2
.6	6	2	7	25	1	1	4
.7	6	3	7	26	0	3	5
.8	7	0	7				
.9	7	1	7				
3.0	7	2	8				
				Drip — --	0	3	5
1 half inch					0	1	1
2 half inch					0	1	0
3 half inch					0	0	5

EXAMPLE

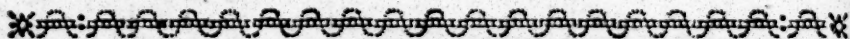
EXAMPLE II.

Suppose there were 15 inches of the copper dry, how many barrels, &c. would there be in it?

OPERATION.

I seek for 15 in the first column of the copper under dry inches, and I find 7 Bs. 2 Fs. 8 Gs. = the content. When the depth is taken to a half inch. then the contents of the half inches under the tables will be of use. But observe when the wet inches are taken, then the content of the half inch must be added; and when the dry inches are taken, the content of the half inch must be subtracted from the content of the even inches.

As for example: Suppose $15\frac{1}{2}$ inches of the copper where dry, how much would it then contain? the content at 15 inches, as found as above, is 7 Bs. 2 Fs. 8 Gs. from which I subtract 0 B. 1 F. 2 G. the area of the 2^d half inch, and there remain 7 Bs. 1 F. 6 Gs. = the content required.



CHAP. XI.

Of cask gaging.

CASK gaging is a very curious art, and is the most difficult part of gaging.

The difficulty arises from the great variety of curves that these kind of vessels are composed of, and for that reason it requires a good deal of judgment, and practice, to determine the exact content of any close vessel. In order to have a right understanding in the matter, it would be necessary for the gager to have some idea of the conic sections, because casks are generally compared to solids generated by one or other of those sections; which would enable him the better to distinguish to which of these solids the cask's curve bears the nearest resemblance.

Gagers have reduced those different curvatures of casks to four varieties, &c.

Var. 1. Those casks that are most curved, or bent, are considered as the middle frustum of a spheroid, which is represented by the outer lines of the following figure, viz. $AIOIBCIAID$.

Var. 2 When the staves are not so much arching, or bent, the cask is supposed to be the middle frustum of a parabolic spindle, which is represented by the lines $A2O2BC2a2D$.

Var. 3. When the staves are but little bent, or curved, they are reputed to be in the form of the frustums of two parabolic conoids joined together at their greatest bases, and is represented by the lines $A3O3BC3a3D$.

Var. 4. When the lines are straight from head to bung, so as at the lines $A4O4BC4a4D$, then it is plain that such cask is formed of the frustums of two cones joined together at their greater bases.

In the practice of cask gaging the officers of the excise are obliged to follow one general method, wherein they suppose that a cask may have the same dimensions with respect to its head, bung, and length, and yet by reason of the great or little archedness of the cask's staves between the head and bung, the cask may belong to either of the four varieties; which is according to Mr. *Everard's* system of cask gaging, and agreeable thereto are the several varieties graduated on the sliding rule, in order to reduce each variety to a mean diameter, or that of a cylinder whose height and capacity is nearly equal to that of the cask.

And as the contents of casks are generally found by the rule, and that being the test whereby all those contents must be tried, I shall all along confine myself to such rules as will produce the same content as though they were cast by the sliding rule; so the young gager may use which method he thinks most proper.

ART. I.

To find the content of a cask in any of the four varieties both by pen and rule.

RULE.

1. To find a mean diameter.

THE casks being reduced to four varieties, if you multiply the difference between the head and bung diameters, when it is less than 6 inches,

$$\text{By } \left\{ \begin{array}{l} ,68 \\ ,62 \\ ,55 \\ ,50 \end{array} \right\} \text{ for the } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{ Variety,}$$

or if the difference between the head and bung exceed 6 inches,

$$\text{By } \left\{ \begin{array}{l} ,7 \\ ,64 \\ ,57 \\ ,52 \end{array} \right\} \text{ for the } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{ Variety,}$$

and add the Product to the head diameter, then that sum will be a mean diameter.

2. To find the content.

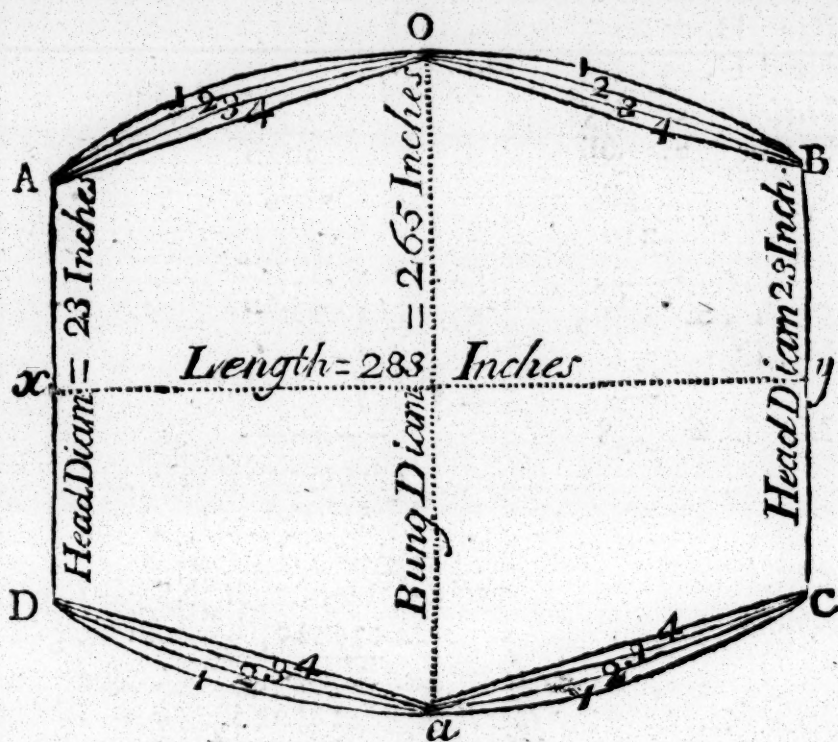
Square the mean diameter, and multiply that square by the length of the cask, and the product multiply or divide by the factors, &c. in page 57, and the product or quotient will be equal to the content of the cask.

Or,

By Art. ix. Chap. vii. find the area of the mean diameter, which area multiplied into the length will be equal to the content of the cask.

The

The figure of the four varieties of casks.



EXAMPLE.

Let the above figure represent all the four varieties of casks, whose bung diameter $o a$ is $= 26.5$ inches, head diameter $A D = B C = 23$ inches, and length $x y = 28.3$ inches; to find the content in ale gallons, supposing it to belong to all the four varieties.

1. For the spheroid, or 1st variety.

Operation by the pen.

Bung D. = 26.5

Head D. = 23.0

Diff. = 3.5

Factor = .68

$\frac{280}{210}$

Prod. = 2.380

Hd. D. = 23.

Mn. D. = 25.38

M. D. } = 25.4
nearly }

25.4

1016

1270

508

Sq. of } 645.16
M. D. }

Length } 28.3
of cask }

193548

516128

129032

359.05) 18258.028 (50.8 = cont.

179525

in ale galls.

.. 305528

287240

Rem. — .18288

By the rule.

By the difference of the diameters, which in this case = 3.5, look on the line of inches on the edge of the rule, and see what number stands against it on the line marked Spheroid, and whatsoever number you find there, add to the head diameter, and the sum will be a mean diameter; as in this example against 3.5 is 2.4 + 23 = 25.4 = mean diameter, as before found.

Then the proportion for the content will be

on D.	on c.	on D.	on c.	
As 18.95	: 28.3	: 25.4	: 50.8	= Cont. in ale
Gage pt.	Length.	Mean D.	Cont.	galls.

Thus

Thus the content found both by pen and rule is the same, viz. 50.8 gallons, which being nearest to 51 may be called 51 gallons; and so much would the cask contain if it were the frustum of a spheroid.

2. For the middle frustum of a parabolic spindle, or that of the 2d variety.

Operation by pen.

$$\begin{array}{l} \text{Diff. of } \} \\ \text{Diams. } \} = 3.5 \\ \text{Factor } = .62 \end{array}$$

$$\begin{array}{l} \text{Mean D. } \} \\ \text{nearby } \} = 25.2 \end{array}$$

$$\begin{array}{r} 70 \\ 210 \\ \hline \end{array}$$

$$25.2$$

$$504$$

$$1260$$

$$504$$

$$\text{Prod.} = 2.170$$

$$\text{H.D.} = 23.$$

$$\begin{array}{l} \text{Sq. of } \} \\ \text{M.D. } \} = 635.04 \end{array}$$

$$\text{M.D.} = 25.170$$

$$\begin{array}{l} \text{Length } \} \\ \text{of cask } \} = 28.3 \end{array}$$

$$\begin{array}{r} 190512 \\ 508032 \\ 127008 \\ \hline \end{array}$$

$$359.05) 17971.632 (50.0 = \text{cont. in ale gallons.}$$

$$179525$$

$$\text{Rem. } \dots 19132$$

By the rule.

With the difference of the diameters look on the line of inches on the edge of the rule, and whatsoever number stands against it on the line marked 2d Variety, add to the head diameter, and that sum will be a mean diameter.

In this case the difference is 3.5, against which on the line marked 2d Variety is 2.2 nearly, which added to the head diameter = 25.2 = the mean diameter before found.

H 2

Then

Then the proportion for the content will be

on D. on C. on D. on C.
As 18.95 : 28.3 :: 25.2 : 50 = as before found.
Gage pt. Length. Mean D. Content.

Thus the content found both by pen and rule is the same, viz. 50 gallons, and so much would the cask contain if it were in the form of a parabolic spindle called the 2d variety.

3. For the frustum of two parabolic conoids, or 3d variety of casks.

Operation by pen.

Diff. of } = 3.5	Mean D. } = 24.9
diams. }	nearly }
Factor = .55	
	24.9
	<u>2241</u>
	996
	<u>498</u>
Prod. = 1.925	
H. D. = 23.	Sq. of } = 620.01
	M. D. }
M.D. = 24.925	Length = 28.3

	186003
	<u>496008</u>
	124002
359.05	17546.283
	<u>143620</u>
	.318428
	<u>287240</u>
	.311883
	<u>287240</u>
Remains	.24643

48.8 = content in ale gallons.

By the rule.

With the difference of diameters look on the line of inches on the edge of the rule, and whatsoever number stands against it on the line marked 3d Variety, add to the head diameter, and that sum will be a mean diameter.

Of Cask gaging.

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In this case the number standing against 3.5 on the line marked 3d Variety is $1.9 + 23 = 24.9$ mean diameter as before found. Then the proportion for the content is,

on D. on c. on D. on c.
As 18.95 : 28.3 : : 24.9 : 49 = as before
Gage pt. Length. M. Diam. Cont. found.

The content found both by pen and rule agree very well, for they both may be called 49 gallons, that being the nearest even gallon; and so much would the cask contain if it were in the form of the frustum of two parabolic conoids, or 3d variety.

4. For the frustum of two cones joined together at their greater bases, called the 4th variety.

Operation by pen.

Diff. of } = 3.5	M. diams. = 24.75
diams. }	24.75
Factor = .5	12375
Prod. = 1.75	17325
H. D. = 23	9900
M. D. = 24.75	4950
Square of mean diameter)	612.5625
	28.3
	18376875
	49005000
	12251250
359.05)	1733551875 (48.281 = cont.
	143620 in ale gallons.
	.297351
	287240
	.101118
	71810
	293087
	287240
	.058475
	35905
Remains —	22570
H 3	

By

By the rule.

With the difference of diameters look on the line of inches on the other edge of the rule, and whatsoever number stands against it on the line marked F c, add to the head diameter, whose sum will be a mean diameter.

In this case the number standing against 3.5 on the line F c is 1.77 nearly, which added to the head diameter = 24.77, the mean diameter.

Then for the content the proportion is

on D.	on C.	on D.	on C.	
As 18.95	: 28.3	: :	24.77	: 48.3 = content.
Ale gage	Length.	M.Diam.	Content.	
point.				

The content found both by pen and rule is nearly alike, and may be called 48, because it is the nearest even gallon.

In like manner might the content of the several varieties have been found in wine measure, if instead of dividing the product of the square of the mean diameter, and the length of the cask by the ale divisor = 359.05, I had made use of 294.12, the wine divisor.

And the proportion by the rule for wine gallons would be, as the wine gage point on D is to the length of the cask on C, so is the mean diameter on D to the content in wine gallons on C.

As for example, If the content of the 1st variety was required in wine gallons, the proportion is

on D.	on C.	on D.	on C.	
As 17.15	: 28.3	: :	25.2	: 61.1 = wine galls.
W. gage	Length.	M.Diam.	Content.	
point.				

And in this manner may the content of the other three varieties be found in wine gallons.

ART. II.

To find the content of a cask supposed to belong to all the four varieties, when the difference of the head and bung diameter is above 6 inches.

EXAMPLE.

LET the cask's head diameter = 24.5 inches, bung diameter = 31.5 inches, and the length = 42 inches : to find the content in ale gallons in all the four varieties.

OPERATION.

Bung diam. = 31.5

Head diam. = 24.5

Diff. diams. = 7.0

Head diam. = 24.5 + $\left\{ \begin{array}{l} 7 \times .7 = 29.4 \\ 7 \times .64 = 28.98 \\ 7 \times .57 = 28.49 \\ 7 \times .52 = 28.14 \end{array} \right\}$ Mean diameters.

The areas $\left\{ \begin{array}{l} 2.407 \\ 2.339 \\ 2.261 \\ 2.206 \end{array} \right\}$ which $\times^d$ $\left\{ \begin{array}{l} 101.09 \\ 98.23 \\ 94.96 \\ 92.65 \end{array} \right\}$ Contents of
of the M. $\left\{ \begin{array}{l} \text{by 42, the} \\ \text{length of} \\ \text{the cask} \end{array} \right\}$ several varieties in ale
Ds. are $\left\{ \begin{array}{l} 2.407 \\ 2.339 \\ 2.261 \\ 2.206 \end{array} \right\}$ the cask = $\left\{ \begin{array}{l} 101.09 \\ 98.23 \\ 94.96 \\ 92.65 \end{array} \right\}$ gallons.

The contents may be found by the sliding rule in the same manner as those were in the last article.

ART. III.

To find the content of all the varieties of casks another way.

EXAMPLE.

LET the cask be supposed to have the same dimensions as that in the last article : to find the content in ale gallons in all the four varieties.

Of Cask gaging.

1. For the spheroid, or 1st variety.

R U L E.

To twice the square of the bung diameter add the square of the head diameter, and multiply this sum by the length of the cask, and the product divide by 1077 ($= 3 \times 359$, the circular divisor for ale gallons in page 57) and the quotient will be = to the content in ale gallons.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Twice the square of the } \} & = & 1984.5 \\
 \text{bung diam. } 31.5 & & \\
 \text{Square of head diam. } 24.5 & = & 600.25 \\
 \hline
 \text{Sum} & - & - & - & = & 2584.75 \\
 \text{When multiply by the length} & = & 42 \\
 \hline
 \end{array}$$

1077)108559.5(100.8=Cont.

If the product of the sum of the squares and length of the cask were divided by 882 = 3 times the circular divisor for wine gallons in page 57, the quotient would have been = the content in wine gallons.

2. For the second variety of casks.

To twice the square of the bung diameter add the square of the head diameter, and subtract four tenths of the square of the difference of the said diameters; multiply the remainder by the length of the cask, divide the product by 1077, and the quotient will be its content in ale gallons.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Twice Sq. of B. D. } 31.5 & = & 1984.5 \\
 \text{Square of H. D. } 24.5 & = & 600.25 \\
 \text{Sum} & - & - & - & = & 2584.75 \\
 \frac{4}{10} \text{ of sq. of diff. of diam. } 19.6 & = & 19.6 \\
 \text{Remainder} & - & - & - & = & 2565.15 \\
 \hline
 & & & & & 42 \\
 \hline
 \end{array}$$

1077)107736.30(100.0 ale gallons.

3. For

3. For the 3d variety of casks.

RULE.

Multiply the sum of the squares of the head and bung diameters by the length of the cask, and divide that product by twice the circular divisor in page 57; the quotient will be the content of the cask, of the same denomination as the divisor made use of.

OPERATION.

$$\text{Square of B. D. } 31.5 = 992.25$$

$$\text{Square of H. D. } 24.5 = 600.25$$

$$\text{Sum of the squares} = 1592.5$$

$$\text{Which } \times \text{ by length} = 42$$

$$\begin{array}{r} 31850 \\ 63700 \\ \hline \end{array}$$

$$718)66885.0(93.2 = \text{cont. in ale gallons.}$$

4. For the 4th variety of casks.

RULE.

From the sum and half sum of the squares of the head and bung diameters subtract half the square of the difference of the diameters; then multiply the remainder by the length of the cask, and the product divide by three times the circular divisor in page 57; the quotient will be equal to the content of the cask of the same denomination as the divisor made use of.

OPERATION.

Square of B. D.	31.5 =	992.25	B. diam. =	31.5
Square of H. D.	24.5 =	600.25	H. diam. =	24.5
Sum of squares		= 1592.50	Diff. of diam.	7.0
$\frac{1}{2}$ Sum of squares		= 796.25		7
Sum	— —	= 2388.75	Sq. of diff.	49
		24.5	$\frac{1}{2}$ Sq. of diff.	24.5
Remainder	— —	= 2364.25		
Which $\times$ by length		42		

1077)99298.5(92.2 = content in
ale gallons.

The several contents thus found agree very well with those found by the former method in the last article; so either rule may serve for the finding of the content of a cask belonging to any of the four varieties by the pen. But the sliding rule is preferable to either, because by that the contents are found with much more ease and expedition, and near enough the truth in practice.

ART. IV.

To find the content of a cask in the form of a spheroid by the lines c and d on the rule, without the help of the lines of varieties.

RULE.

SET the length of the cask on c.

to $\left\{ \begin{array}{l} 32.8 \\ 29.7 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Ale} \\ \text{Wine} \end{array} \right\}$ Gallons on d.

Then seek for the bung diameter on d, and note the number against it on c, which double; then seek the head diameter on d, and note the number against it on c,

c, which number added to double the other before found, will = the content of the cask in ale or wine gallons respectively.

EXAMPLE.

Let the dimensions of the cask be the same as that in the two last articles, viz. head diameter = 24.5, bung diameter = 31.5, and the length = 42 inches: to find the content both in ale and wine gallons.

1. For the content in ale gallons.

Set 42 = the length on c to 32.8 on D, then against the bung diameter = 31.5 on D is 38.7 on c, which doubled = 77.4; then seek for 24.5 = the head diameter on D, and against it is 23.5 on c, and $77.4 + 23.5 = 100.9$ = the content of the cask in ale gallons, which is very near the same as found by the former rules.

2. For the content in wine gallons.

Set 42 = the length on c to 29.7 on D, then against the bung diameter 31.5 on D is 47.2 on c, which doubled = 94.4; next find the head diameter = 24.5 on D, and against it is 28.7 on c, and $94.4 + 28.7 = 123.1$ = the content in wine gallons.

This is a very useful rule for finding the contents of the 1st variety of casks, because it may be performed in less time this way than by the line of varieties on the rule.

ART. V.

To find the content of a cask without regarding the variety at all, by having a fourth dimension given that is a diameter in the middle between the head and bung of the cask.

RULE.

SET the length of the cask on c.

to $\left\{ \begin{array}{l} 46.4 \\ 42.0 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Ale} \\ \text{Wine} \end{array} \right\}$ Gallons on D.

H 6

Then

Then find the head, bung, and middle diameters on D , and note the numbers against them on C ; then, if to the sum of the first and second of these numbers there be four times the third added, that sum will be equal to the content of the cask in ale or wine gallons. There is one difficulty arising from this rule, viz. the finding of a diameter in the middle between the head and bung of the cask. But this may be effected by a plumb line, according to the following directions.

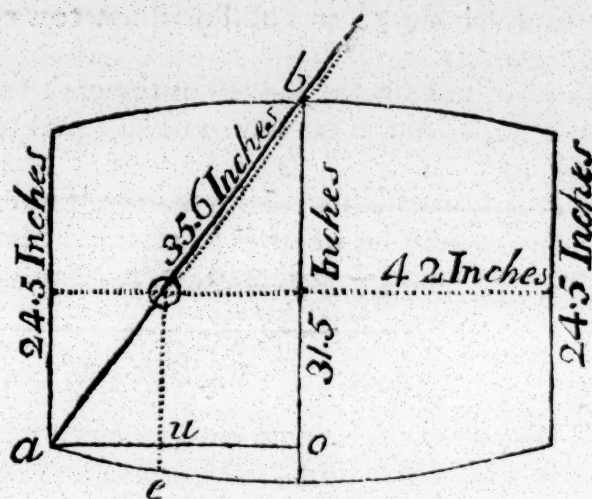
1. Get a ring just to fit your gaging rod, so as to slide up and down at pleasure; file a small notch in the inside of the ring for the line to move freely up and down with the weight of the plummet.

2. Put your gaging rod into the bung hole of the cask, and measure the diagonal ba , (see the following figure) which suppose = 35.6 inches; then take out the rod again, and fix the ring on it at 17.8, equal to half the diagonal, as at $\odot$, and put the gaging rod again into the cask, with the plumb line running through the notch of the ring till it just touch the side of the cask at e ; then holding the thread close to the rod, take it out of the cask and measure the distance $\odot e$, that is the length of the line from the point of suspension to the extremity of the plummet, which suppose = 16.7. This reserve.

* 3. From half the bung diameter subtract one fourth part of the difference between the head and bung diameters. The remainder subtract from the reserved number, and this last remainder double and add to the head diameter, and the sum will be equal to the middle diameter required.

* The triangles $ab\odot$ and $a\odot u$ being similar, therefore when $b\odot$ is half of ab , $a u$ will be half of $a\odot$, let the bung diameter = B , the head diameter = H , $\frac{1}{2}$ the length = L , and the difference of diameters = X ; then $\frac{1}{2}B - \frac{1}{4}X :: \frac{1}{2}L : \frac{1}{2}B - \frac{1}{4}X = \odot u$, and $\odot e - \odot u = ue$, and $2ue + H$ = the middle diameter.

EXAM-



EXAMPLE.

Suppose the head diameter of the cask above = 24.5 inches, bung diameter = 31.5, and length = 42 inches; to find the content in ale gallons.

OPERATION.

$\frac{1}{2}$ Bung diameter 31.5 = 15.75	B. diam. = 31.5
$\frac{1}{4}$ of diff. of diameters = 1.75	H. diam. = 24.5
1st Remainder — = 14.00	Diff. — = 7.0
Length $\odot$ e found = 16.7	$\frac{1}{4}$ Diff. = 1.75
2d Remainder — = 2.7	
	2.7
Double the last rem. = 5.4	
Head diameter — = 24.5	
Middle diam. — = 29.9	

Then for the content.

Set 42 on c to 46.4 on D , then against the head diameter 24.5 on D is 11.73 on c , against the bung diameter 31.5 is 19.4; and against the middle diameter 29.9 is 17.44, which multiplied by 4 = 69.76, and

11.73

$11.73 + 19.4, + 69.76 = 100.89 =$ the content of the cask in ale gallons.

If, instead of setting the length of the cask to 46.4, I had set it to 42, then the content would have been found in wine gallons.

This rule is universal for all sorts of bulged casks, let their form or variety be of any kind whatsoever. See Mr. Shirtcliff's Gaging, page 220, &c.

ART. VI.

To find the content of a spheroidal cask, by having the length of the diagonal line given, without knowing the dimensions of the head, bung, and length at all.

1. By the Branan's rule.

PUT the Branan's rule into the bung hole of the cask (supposing it to be in the middle of the cask) till the end of it meet the intersection of the head, and the staves on the opposite side; then on that part of the rule held in the center of the bung hole on the diagonal lines is the content in ale and wine gallons. This is so easy that it requires no example.

2. By the lines D and E on the rule.

Set 30 on D

to $\left\{ \begin{array}{l} 60 \\ 73.3 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Ale} \\ \text{Wine} \end{array} \right\}$ Gallons on E

Then against any diagonal on D is the content upon E in ale or wine gallons respectively.

EXAMPLE.

Suppose the length of the diagonal of a cask = 28.8 inches, what will the content be both in ale and wine gallons?

1. Proportion for ale gallons.

on D.	on E.	on D.	on E.
As 30	: 60	: : 28.8	: 53 = ale gallons.
Diag.	Cont.	Diag.	Cont.

2. Pro-

2. Proportion for wine gallons.

on D. on E. on D. on E.
 As 30 : 73.3 : : 28.8 : 65 = wine gallons.
 Diag. Content. Diag. Content.

3. By the lines D and c on the rule.

Set the length of the given diagonal of the cask on c
 to $\left\{ \begin{smallmatrix} 21.2 \\ 19.2 \end{smallmatrix} \right\}$ for $\left\{ \begin{smallmatrix} \text{Ale} \\ \text{Wine} \end{smallmatrix} \right\}$ gallons on D

Then against that diagonal on D is the content upon c
 in ale or wine gallons respectively.

EXAMPLE.

Let the diagonal = 28.8 inches as before : to find the
 content of the cask in ale and wine gallons.

Proportion for ale gallons.

on D. on c. on D. on c.
 As 21.2 : 28.8 : : 28.8 : 53 = in ale gallons.
 Gage pt. Diag. Diag. Content.

2. Proportion for wine gallons.

on D. on c. on D. on c.
 As 19.2 : 28.8 : : 28.8 : 65 = wine gallons.
 Gage pt. Diag. Diag. Content.

Thus the contents found either way are the same ; as
 also by the Branan's rule ; for against 28.8 on the line
 of inches there is placed.

53. Ale } gallons
 65. Wine }

on the diagonal line, equal to the contents before found.

ART. VII.

*To find the content of a cask in the form of the frustum of a
 Cone.*

RULE.

WITH some convenient instrument take the length
 of the cask within the heads, and also the top
 and

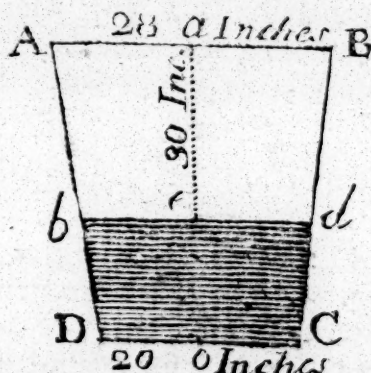
and bottom diameters ; then by Art. iv. Chap. viii. find the content.

Or add the top and bottom diameters together, and take half that sum for a mean diameter, equal to that of a cylinder of the same height as the cask ; then by Art. ii. Chap. viii. find the content, which will be near enough in practice.

EXAMPLE.

Let the following figure represent the cask, whose top diameter $AB = 28$ inches, bottom diameter $DC = 20$ inches; and the length $oa = 30$ inches ; to find the content in ale gallons.

A CONICAL CASK.



OPERATION.

By the rule.

$$\begin{array}{rcl}
 AB & = & 28 \quad \text{on D. on c. on D. on c.} \\
 DC & = & 20 \quad \text{As } 18.95 : 30 :: 24 : 48.1 = \text{ale gs.} \\
 & - & \text{Ale gage Lth. Mean. Content.} \\
 \text{Sum} & = & 48 \quad \text{point.} \\
 \frac{1}{2} \text{ Sum} & = & 24 = \text{Mean.}
 \end{array}$$

The content thus found is $= 48.1$ ale gallons, which, with the mean diameter and length, I have put into the specimen of a dimension book exhibited in Chap. xiii.

CHAP. XII. .

Of ullaging.

DEFINITION.

ULLAGING of casks is the finding of the vacuity of a cask that is part full and part empty: or, which is the same thing, the finding of the quantity of liquor that is in such cask.

ART. I.

To ullage a lying cask by the line of segments on the rule, having the bung diameter, wet inches, and the content of the cask given.

RULE.

1. **S**ET the bung diameter on *c* to 100 on the line marked *S. ly.* or Segments lying: then seek the wet inches on *c*, and observe what number stands against it on the segments, which call a fourth number.

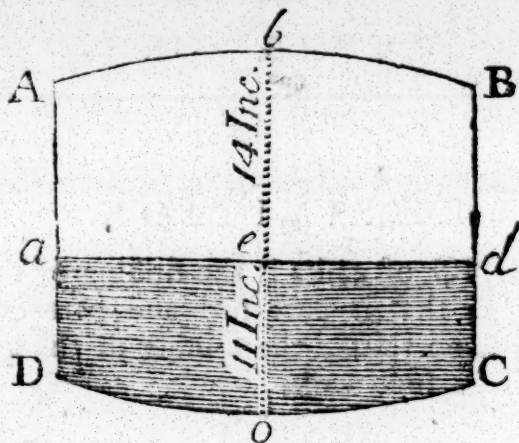
2. Set 100 on *A* to the content of the cask upon *B*, then against the fourth number before found on *A* is the quantity of liquor in the cask on *B*.

EXAMPLE.

Let the following figure represent the lying cask, whose bung diameter = *bo* = 25 inches, and wet inches = *eo* = 11 inches, and content = 48.3 gallons: to find the quantity of liquor in it, or the part *aedcdo*.

A LYING

A LYING CASK.



PROPORTION.

- on c. on S. l. on c. on S. l.
 1. As 25 : 100 :: 11 : 41.4 = 4th number.
 B. diam. Segs. W. inch. Seg.

- on A. on B. on A. on B.
 2. As 100 : 48.3 :: 41.4 : 20 = quantity of
 Segment. Cont. Segment. Cont. liq. in cask.

But if you wanted to know the vacuity of the cask, or what it wanted of being full, viz. the part $A b B d e a$, then, instead of making use of the wet inches, use the dry inches.

Thus,

- on c. on S. l. on c. on S. l.
 1. As 25 : 100 :: 14 : 58.2 = 4th num.
 B. diam. Segs. Dry inch. Segs.

- on A. on B. on A. on B.
 2. As 100 : 48.3 :: 58.2 : 28.1 = Vac. of
 Segment. Cont. Segmt. Vac. cask.

This last sum found added to the quantity of liquor in the cask, viz. $20 + 28.1 = 48.1$, which is but two tenths less than the content of the cask.

ART.

ART. II.

To find the ullage of a standing cask by the line of segments on the rule, having the length, wet inches, and the content of the cask given.

RULE.

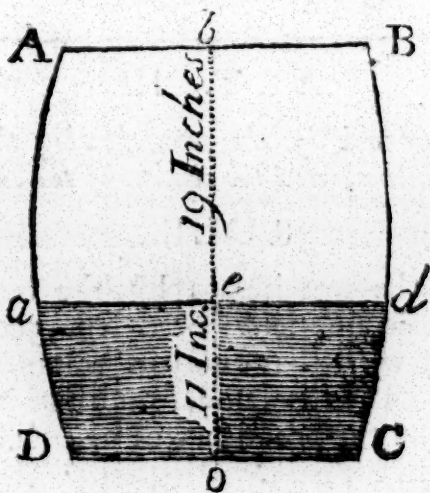
1. SET the length of the cask on c to 100 on the line marked ff. or ft. or segments standing; then seek the wet inches on c, and observe what number stands against it on the segments, which call a fourth number.

2. Set 100 on A to the content of the cask upon B, then against the fourth number before found on A is the quantity of liquor in the cask upon B.

EXAMPLE.

Let the following figure represent the standing cask, whose length $bo = 30$ inches, and wet inches $eo = 11$ inches, and content = 48.3 gallons; to find the quantity of liquor in it, or the part $aedc$ o D.

A STANDING CASK.



OPERATION.

on c.	on ff.	on c.	on ff.
1. As 30	: 100	: 11	: 35.2 = 4th numb.
Length.	Segmts.	W. inches.	Segmts.

2. As

- | | | | | |
|-----------|--------|----------|-------|---------------|
| on A. | on B. | on A. | on B. | |
| 2. As 100 | : 48.3 | : : 35.2 | : 17 | = quantity of |
| Segment. | Cont. | Segment. | Cont. | liq. in cask. |

But to find the vacuity of the cask, or the part $A b B d e a$, the proportion will stand thus:

- | | | | | |
|----------|--------|----------|--------|-------------|
| on c. | on ff. | on c. | on ff. | |
| 1. As 30 | : 100 | : : 19 | : 64.3 | = 4th numb. |
| Length. | Segmt. | D. inch. | Segmt. | |

- | | | | | |
|-----------|--------|----------|-------|--------------|
| on A. | on B. | on A. | on B. | |
| 2. As 100 | : 48.3 | : : 64.3 | : 31 | = vacuity of |
| Segmt. | Cont. | Segmt. | Cont. | cask. |

The quantity of liquor in the cask = $17 + 31$ = the vacuity = 48 gallons, nearly equal to the content of the cask.

The difference between the sum of the seperate parts thus found, and the whole content of the cask, is occasioned by the line of segments being adapted to one particular sort of casks only; which is not very material, and may do well enough in practice.

ART. III.

To ullage a lying cask by the pen, having the bung diameter, wet inches, and the content of the cask given.

RULE.

1. **D**IVIDE the wet inches by the bung diameter, and if the quotient be under ,500 subtract a fourth part of what that quotient wants of ,500 from the quotient, and the remainder multiply by the content of the cask, and the product will be equal to the quantity of liquor in the cask.

2. When the quotient of the wet inches divided by the bung diameter exceeds ,500, then add a fourth part of that excess to the quotient, and that sum multiplied by the content of the cask will produce the content of the liquor in the cask.

EXAM-

EXAMPLE.

Let the dimensions and content of the cask be the same as in Art. i. of this chapter, viz. bung diameter = 25 inches, wet inches = 11, and the content of the cask = 48.3 gallons: to find the quantity of liquor in the cask.

OPERATION.

B. D. W. inc. Quotient.

25) 11.0 (,44 = less than ,500.

,500

,060 = difference.

,015 = $\frac{1}{4}$ of diff. subtract.

,425 = multiplier.

48.3 = content of the cask.

1275

3400

1700

20,5275 = content of the liquor in cask.

The vacuity of the cask may be found by dividing the dry inches by the bung diameter, instead of the wet, and managing the quotient according to the foregoing rules.

The content of the ullage found by the pen differs somewhat from that found by the line of segments on the rule. Which is nearest the truth I will not take upon me to determine; however, the line of segments must be made use of by the officers of the excise for computing the ullages of such casks.

ART. IV.

To find the content of the ullage of a standing cask by the pen, having the length, wet inches, and the content of the cask given.

RULE.

1. **D**IVIDE the wet inches by the length of the cask, and if the quotient be under ,500, subtract one tenth part of what the quotient wants of ,500 from the quotient.

2. If the quotient exceeds ,500, add one tenth part of that excess above ,500 to the quotient, then if the difference in the first case, or the sum in the second be multiplied by the content of the cask, the product will be equal to the quantity of liquor in the cask.

EXAMPLE.

Let the dimensions and content of the cask be the same as in Art. ii. of this chapter, viz. length = 30 inches, wet inches = 11, and the content of the cask = 48.3 : to find the quantity of liquor in the cask.

OPERATION.

Lth. W.inch. Quotient.

$$30 \) \ 11.0 \ (\ ,367 \text{ fere} = \text{less than } ,500.$$

,500

—
,133 = difference.

—
,0133 = $\frac{1}{8}$ of diff. subtract.

—
,3537 = multiplier.

48.3 = content of the cask.

—
10611

28296

14148

—
17,08371 = content of liquor in cask.

This

This agrees very well with the ullage found by the line of segments in Art. ii. of this chapter. If you make use of the dry inches instead of the wet, and manage the quotient as above, you will find the vacuity of the cask = 31.21629 gallons.

ART. V.

To ullage a cask in the form of the frustum of a cone. See the figure in Art. vii. Chap. xi.

RULE.

TAKE the top and bottom diameters, wet inches, and the length of the cask; then by Art. x. Chap. viii. find the diameter of the cask at the liquor's surface; then having the top and bottom diameters, and the height of the little frustum *bedc* *oD*, the content may be found by Art. iv. Chap. viii. which will be equal to the quantity of liquor in the cask. But in practice you need only take the wet inches, and guess at a mean diameter by laying the gaging rod over the top of the cask; and then find the content by the rule in the same manner as if it were a cylinder.

EXAMPLE.

Suppose the depth of the liquor = *eo* = 12.5 inches, and I found the mean diameter = 22 inches, what is the content in ale gallons?

PROPORTION.

on D.	on C.	on D.	on C.	
As 18.95	: 12.5	: : 22	: 16.9	= ale gallons.
A. gage pt.	W. inc.	M. D.	Cont.	

CHAP. XIII.

Of gaging of malt, &c.

ART. I.

To gage and fix a maltster's square, or oblong cistern.

RULE.

1. **W**ITH a dimension cane, or some other convenient instrument, measure the length and breadth of the cistern in several places, and in case you find any variation, add the lengths or breadths together, and divide their sum by the number of dimensions taken of each, and the quotient will be a mean length or breadth; and at the same time take also the depth of the cistern.

2. To find the area of the cistern, multiply the length by the breadth, and that product multiply or divide by the factors, &c. for square malt bushels in page 57; and the product or quotient will be equal to the area, which, with the dimensions, transfer to the specimen of a dimension book, exhibited in Art. ix. of this chapter.

EXAMPLE.

Suppose the length of the cistern was found to be = 114 inches, the breadth = 58.5 inches, and the depth = 36 inches; to find the area at one inch, and fix it in the dimension book.

OPERATION.

Length = 114

Breadth = 58 5

570

912

570

2150.42)6669.00(3.10 = area in bushels.

645126

.217740

215042

Rem.

.. 26980

It will be necessary to prove your work by the rule, in order to prevent any mistake that may happen in the operation by the pen; and for that purpose the proportion will stand thus:

on A.	on B.	on A.	on B.	
As 2150.42	: 114	: 58.5	: 3.10	as before.
M. B.	Length.	Breadth.	Area.	

The answer comes out the same both by pen and rule, which proves the work to be right: therefore I transfer the same, with the dimensions, to the specimen of a dimension book in Art. ix. of this chapter, which was required.

Note, If any depth taken in this cistern be multiplied by the area, the product will be equal to the content of the malt that is in it.

ART. II.

To gage and fix a maltster's round cistern, whether it be the frustum of a cone, or part of a hoghead or pipe standing upon one head, the other being cut off.

RULE.

1. **W**ITH some convenient instrument take mean diameters between every six or ten inches of the depth, according to the greater or lesser difference of the diameters of the vessel, as was shewn in fixing of victuallers utensils. At the same time take the depth.

2. To find the area of each mean diameter, square the diameters, and multiply or divide each square by the circular factors, &c. for malt bushels in page 57; and the products or quotients will be equal to the several areas required.

EXAMPLE.

Suppose the depth of the cistern, or uting fat, to be gaged, &c. = 30 inches, and the diameter at 5 inches from

from the base = 29.6 inches, at 15 inches from the base the diameter = 33 inches and at 25 inches from the base the diameter = 36.2 inches; to find the respective areas of the mean diameters in malt bushels.

O P E R A T I O N.

1 Mean diameter = 29.6

29.6

—

1776

2664

592

2738.)876.16(.32 = lower area.

8214

—

.5476

5476

Rem. ...0

In like manner is the area of 33 inches found to be = .40, and the area of 36 inches = .48, as I have set them in the specimen of a dimension book exhibited in the ninth article of this chapter.

The proportion by the rule for the areas is,

on D. on C. on D. on C.

As $\left\{ \begin{array}{l} 52.32 : 1 :: 29.6 : .32 = 1^{\text{st}} \text{ area.} \\ 52.32 : 1 :: 33 : .40 = 2^{\text{d}} \text{ area.} \\ 52.32 : 1 :: 36.2 : .48 = 3^{\text{d}} \text{ area.} \end{array} \right.$

C. G pt. Unity. M. D. Area.

Note, All depths that are taken in this cistern must be multiplied by the respective areas to which they belong, viz. If 10 inches or under by the first area, that part from 10 to 20 by the second area, and from 20 to 30 inches of the depth by the third area, which added together, will be the content at any depth.

ART. III.

To gage a couch of malt in a square or oblong frame, and find the content of the same.

RULE.

1. **W**ITH a box and tape measure the length and breadth of the frame, and with a gaging-rod take the depth of the malt in several places; then add these depths together, and divide their sum by the number of depths taken, and the quotient will be a mean depth.

2. To find the content multiply the length, breadth, and depth together, and that product divide by the square divisor for malt bushels in page 57, and the quotient will be equal to the content of the malt in the couch.

EXAMPLE.

Suppose the length of a couch of malt was found = 105 inches, the breadth = 104 inches, and the depth = 20 inches; to find the content in bushels.

OPERATION.

Length = 105

Breadth = 104

420

1050

10920

20

Product = 218400

PRODUCT.

2150.42)218400.00(101.5 = cont. in bushels.

215042

.. 335800

215042

1207580

1075210

Remains 132370

I a

To

To find the content by the rule, the proportion is

On M.	on B.	on A.	on B.	
As 20	: 105	:: 104	: 101.5	= as before
Depth.	Length.	Breadth.	Cont.	found.

The common method of gaging malt in frames is to find the area of the frame at one inch deep, and to keep that area fixed in the malt book. Then the content of the couch is easily found; for if you multiply that area by any depth, the product will be the content.

ART. IV.

To gage a couch of malt that is not in a frame, but laid in a square or oblong form, with one or more if its sides slanting.

RULE.

1. IF the couch is laid in the middle of the floor with both the longest and shortest sides slanting, then fasten the tape at the edge of the top surface, and draw out the tape till it comes perpendicular over the outside edge of the bottom on the other side, which may be found by dropping of a few corns of malt, and extending the tape to the place where the corns fall from, so as to touch the outside edge of the bottom.

This do for both length and breadth, which distances will be equal to the true length and breadth; for the slanting part that is cut off on one side; will just fill up the empty space on the other side.

2. If one of the sides, or one of the ends, are laid against a wall that is perpendicular to the horizon, then fix the end of the tape close to the wall on the surface of the malt, and draw out the other end till it comes perpendicular over the outside edge of the bottom; there lay hold of the tape with your finger and thumb, and carry it back again till it just touches the outside edge of the top surface; with the other hand draw the tape right over the malt, and observe the number

her of inches at the place of its doubling, which will be equal to the length or breadth of the couch.

3. Take the depth and find the content according to the directions in the last article, which is so easy that I shall pass over this article without giving any example.

A R T. V.

To gage a couch of malt in the form of the frustum of a cone.

IN some countries there are private maltsters that steep so small a quantity as four or five bushels of barley; and it is the common way of such maltsters to empty their cisterns in the middle of the floor by throwing it all on a heap, whereby there is formed a little cone, and in that posture they leave it for the gager to take his dimensions.

R U L E.

With the gaging-rod strike off the top of the cone, and make it level on the top; then take the depth of the couch in the middle, and fix the end of the tape on the outside edge of the top diameter, and draw out the other end across over the malt till it reaches perpendicularly over the outside edge of the bottom diameter, then that distance will be equal to half the sum of the top and bottom diameters, which may be taken for a mean diameter, though not exactly true; yet it may serve well enough in practice for such small quantities of malt.

2. To find the content of the couch, square the mean diameter thus found, and multiply that square by the depth of the malt, that product divide by the circular divisor for malt bushels in page 57, and the quotient will be the content.

But this may be found near enough the truth by the rule, and with much more ease and expedition, therefore I shall omit the working of this example by the pen.

EXAMPLE.

Suppose the depth of the couch was found to be = 6 inches, and the mean diameter = 50 inches, what will its content be in bushels?

Proportion by the rule.

on D.	on C.	on D.	on C.	
As 52.32	: 6	: : 50	: 5.5	= cont. in bushels.
Gage pt.	Depth.	M. D.	Content.	
M. R.				

ART. VI.

To gage a floor of malt, and to find the content of the same.

1. **F**LOORS of malt are generally laid in a square or oblong form, with their surfaces uneven, that is, in some places deeper than others; therefore with a gaging-rod and brass plate take the depth in divers places, which several depths add together and their sum divided by the number of depths taken, the quotient will be equal to a mean depth. Then with a box and tape measure the length and breadth of the malt, and enter the dimensions in your book.

2. To find the content of the floor of malt: this may be found in the same manner as the couch in Art. iii. of this chapter, both by pen and rule, and if the malt line marked M. D is very good, by the rule itself, if the amount of the floor be under 200 bushels. But if it is not true, and the floor be large, it will be best to use both pen and rule for finding its content according to the following directions.

CASE I.

If the length or breadth of the floor be exactly 215 inches, then multiply the other dimension by the depth, and the product will be the content in bushels. But you must mind to prick off one figure for a decimal when there is none in the depth; and if there is a decimal taken

taken in the depth, then prick off two figures in the product for decimals.

CASE II.

If the breadth of the floor be above 215 inches, then subtract 215 from it, then the remainder reserve. Next multiply the length by the depth of the floor, and prick off one or two decimals from the product, as in the first case.

To this product add the content of a floor whose dimensions are equal to the length and depth of the given floor, and breadth equal to the reserved number, or equal to the excess of the floor's breadth above 215 (which must be found by the rule) and the sum will be equal to the content of the floor in bushels, &c.

CASE III.

If the breadth of the floor be less than 215 inches subtract it from 215, and reserve the remainder; then, as in the last case, multiply the length by the depth, and prick off the decimals by the first case; then from the product subtract the content of a floor whose dimension, are equal to the length and depth of the given floor, and the breadth equal to the reserved number, or equal to what the breadth wants of 215 (which is found by the rule) and the remainder will be equal to the content of the floor in bushels, &c.

EXAMPLE I.

Suppose with a gaging-rod and brass plate I took ten depths, and they were as they stand in the margin, whose sum = 35 inches, which divided by 10 = 3.5 = the mean depth; and suppose the length of the floor = 650 inches, and its breadth = 250 inches, what will the content be in malt bushels?

1	=	3.2
2	=	3.6
3	=	2.5
4	=	3.0
5	=	4.0
6	=	4.5
7	=	3.7
8	=	3.5
9	=	4.1
10	=	2.9

Sum = 35.0
 $\frac{1}{10} = 3.5$

Of Malt gaging.

OPERATION.

Length.	Breadth.	Depth.
---------	----------	--------

650	250	3.5
-----	-----	-----

3.5	215	
-----	-----	--

3250		
------	--	--

1950		
------	--	--

35 = remainder above 215.

Prod. = 227.50

Add = 37.1

 Cont. = 264.6 bushels.

Then by the rule.

on M. D. on B. on A. on B.

 As 3.5 : 650 :: 35 : 37.1 = content to be
 Depth. Length. Diff. Content. added.

As the rule now stands you may prove your work ;
 for if, instead of looking against 35 on A, you look
 against 250 = the breadth on A, you will find 264.6
 on B = content before found, which proves the work
 to be done right.

EXAMPLE II.

Let the length of a floor = 700 inches, breadth =
 200 inches, and the depth = 4 inches : to find the
 content in bushels.

OPERATION.

Length.	Breadth.	Depth.
---------	----------	--------

700	200	4
-----	-----	---

4	215	
---	-----	--

 Product = 280.0

 Sub. = 19.5

 Cont. = 260.5

15 = diff. less than 215.

Then

Then by the rule.

on M. D.	on B.	on A.	on B.	
As 4	: 700	: : 15	: 19.5	= cont. to be sub-
Depth.	Length.	Diff.	Content	tracted.

As the rule now stands, against 200 on A is 260.5 on B equal to the content in bushels, as above found which proves the work to be done right. I have given no example in the first case, because it is so easy that it would be of no use.

ART. VII.

To find the content of a floor of malt (having the dimensions given) without either pen or rule.

1. IF either the length or breadth = 215 or 21.5 the other is the area, allowing one figure in the first, and two in the second case for decimals.

But if neither the length or the breadth be equal to one of the above numbers, but agree with any of the following numbers, viz.

If the length or breadth = }	430 or 43	then mul- tiply the other di- mension by	2	and the product will be the area
	645 or 64.5		3	
	860 or 86		4	
	107.5		5	
	129		6	
	150		7	
	172		8	
	1935		9	

Which area multiplied into the depth will produce the content of the floor, allowing for decimals in the product, according to the foregoing directions.

These numbers are multiples of 215, viz. 430 = 215 × 2, and 645 = 215 × 3, &c. therefore may easily be remembered; and thereby the content of any floor of malt may be found almost by inspection.

15

2. If

2. If it should happen that neither the length nor the breadth agree with any of the foregoing numbers, but are nearly equal to one another, then you may take from one of them as many as will make the other equal to one of the foregoing numbers, and multiply the remainder by the multiplier belonging to the number it was compared to, and the product will be equal to the area, as before.

3. If the length be double the breadth, what you take from the length must be but half added to the breadth; or if you take from the breadth and add to the length, add double the number to the length of what you took from the breadth, and so in proportion for any other.

ART. VIII.

To find the area of a floor of malt, by having the roots of the length and breadth; and also the content, having its depth in inches given.

IN order to find the roots of the length and breadth you must have the root of every bushel, and every tenth of a bushel, marked on the tape; which may be done by the help of the following table of roots; for if at the inches and tenths, as they stand in the column of inches, you mark the bushels and tenths, as they stand against them in the column of roots on the tape, you may thereby find the root of any length or breadth from 23.2 inches to 700.2 inches.

The table is constructed in the following manner; the square root of 2150.42, the solid inches in a bushel of malt, is = 46.37, which is also = to the root of 1 bushel. Now if this number be doubled and trebled, &c. you will also double and treble the roots; for if at 46.37 is the root of one bushel, then at $46.37 \times 2 = 92.7$ is the root of 2 bushels, and at $46.37 \times 3 = 139.1$ is the root of 3 bushels; and thus, by continually adding

ing of 46.37 for every bushel, you may gain as many roots as you please of the even bushels, and the roots of the tenths of a bushel are gained by adding and subtracting one tenth of 46.37 to, and from that sum for every tenth of a bushel.

EXAMPLE.

$$\begin{array}{ll} \text{Root of 1 bush.} = 46.37 & \text{Root of 1 bush.} = 46.37 \\ \frac{1}{10} \text{ of } 46.37 \text{ add} = 4.637 & \frac{1}{10} \text{ of } 46.37 \text{ sub.} = 4.637 \end{array}$$

$$\begin{array}{ll} \text{Root of 1.1 B.} = 51.007 & \text{Root of .9 B.} = 41.733 \\ \quad \quad \quad 4.637 & \quad \quad \quad 4.637 \end{array}$$

$$\begin{array}{ll} \text{Root of 1.2 B.} = 55.644 & \text{Root of .8} = 37.096 \\ \quad \quad \quad 4.637 & \quad \quad \quad 4.637 \end{array}$$

$$\begin{array}{ll} \text{Root of 1.3 B.} = 60.281 & \text{Root of .7} = 32.459 \end{array}$$

And thus, by continuing the work, was the following table of roots completed.

Now by having these roots marked on the backside of the tape, as they stand in the table, it will be an easy matter to find the area of any floor of malt.

For if the roots of the length and breadth of the floor are taken with a tape thus marked, and multiplied together, the product will be the area of that floor; which area multiplied by the depth will produce the content; which may be performed by the head without either pen or rule.

EXAMPLE. I.

Suppose I found the root of the length of a floor 10.2 bushels, and the root of the breadth = 3.3 bushels, and the depth = 4 inches; to find the content.

$$\begin{array}{l} 10.2 \times 3.3 = 33.66 = \text{area, which multiplied by 4} \\ 134.6 = \text{contents in bushels.} \end{array}$$

A Table of Roots.

Inc.	Rts.	Inc.	Rts.	Inc.	Rts.	Inc.	Rts.	Inc.	Rts.
23,2	5	162,3	5	301,4	5	440,5	5	579,6	5
27,8	6	166,9	6	306,	6	445,2	6	584,3	6
32,5	7	171,6	7	310,7	7	450,	7	588,9	7
37,1	8	176,2	8	315,3	8	454,4	8	593,	8
41,7	9	180,8	9	320,	9	459,1	9	598,2	9
46,4	10,B.	185,5	10,B.	324,6	10,B.	463,7	10,B.	602,8	10,B.
51,	11	190,1	11	329,2	11	468,3	11	607,4	11
55,6	12	194,8	12	333,9	12	473,	12	612,1	12
60,3	13	199,4	13	338,5	13	477,6	13	616,7	13
64,9	14	204,	14	343,1	14	482,2	14	621,4	14
69,6	15	208,7	15	347,8	15	486,9	15	626,	15
74,2	16	213,3	16	352,4	16	491,5	16	630,6	16
78,8	17	217,9	17	357,	17	496,2	17	635,3	17
83,5	18	222,6	18	361,7	18	500,8	18	640,	18
88,1	19	227,2	19	366,3	19	505,4	19	644,5	19
92,7	20,B.	231,8	20,B.	371,	20,B.	510,1	20,B.	649,2	20,B.
97,4	21	236,5	21	375,6	21	514,7	21	653,8	21
102,	22	241,1	22	380,2	22	519,3	22	658,3	22
106,7	23	245,8	23	384,9	23	524,	23	663,1	23
111,3	24	250,4	24	389,5	24	528,6	24	667,7	24
115,9	25	255,	25	394,1	25	533,3	25	672,4	25
120,6	26	259,7	26	398,8	26	537,9	26	677,	26
125,2	27	264,3	27	403,4	27	542,5	27	681,6	27
129,8	28	268,9	28	408,1	28	547,2	28	686,3	28
134,5	29	273,6	29	412,7	29	551,8	29	690,9	29
139,1	30,B.	278,2	30,B.	417,3	30,B.	556,4	30,B.	695,5	30,B.
143,7	31	282,9	31	422,	31	561,1	31	700,2	31
148,4	32	287,5	32	426,6	32	565,7	32		
153,	33	292,1	33	431,2	33	570,4	33		
157,7	34	296,8	34	435,9	34	575,	34		

ART. IX.

A Table of the dimensions, and content in cubic inches, of the standard bushel, half bushel, peck, and gallon; with the avoirdupoise weight of water that each will contain.

A Table.

Names.	Diam.	Depth.	Cubic inches.	Avoirdupoise.		
				lb.	oz.	drs.
Bushel.	18,5	8,0	2150,42	77	12	8,60
Half bushel.	14,7	6,34	1075,21	38	14	4,30
Peck.	11,7	5,0	537,60	19	7	2,15
Gallon.	9,3	3,96	268,80	9	11	9,07

By this table it will be easy to make any of the measures therein mentioned, to try such as are already made by a good pair of scales.

N. B. It is not material whether or no the dimensions of the bushel be the same with the standard, provided the capacity of it be the same both in weight and measure.

A Specimen of a Dimension Book.

40,0 1 MT	30,0 2 MT	38,0 Cop.	Area.	10,0 ⊙ U. B.	Back.	40,0 1 Rd	40,0 2 Rd	Area.
60,2	60,0 60,0	50,0 48,5 46,5	6,96 6,55 6,02	T 72,0 C 50,0	L 91,3 B 59,7	50,0 ---	67,5 62,6 57,7	12,69 10,91 9,27
12,54	15,86	44,3	5,46	10,03	19,33	6,96	52,8	7,76

24,0 3 Rd	Area.	30,0 4 R.	⊙ Area.		30,0 1. Sq.	Area.
24,9	1,73	T 44,4	4,75	Sq. {	42,5	6,41
26,5	1,96	C 38,4			42,5	
25,0	1,74	T 43,4	4,41	Sq. {	38,5	5,25
24,0	1,60	C 37,5			38,5	
		T 42,2	4,02	Sq. {	32,5	3,74
		C 34,2			32,5	

The Dimensions, &c. of Casks.

No.	H.	B.	L.	Con.	No.	H.	B.	G.	Con.
1	23,0	26,5	28,3	51	2 V.				
	2 V.				6	24,5	31,5	42,0	98
2	23,0	26,5	28,3	50	3 V.				
	3 V.				7	24,5	31,5	42,0	95
3	23,0	26,5	28,3	49	4 V.				
	4 V.				8	24,5	31,5	42,0	92
4	23,0	26,5	28,3	48	9 Mn.	24,0	30,0		48
5	24,5	31,5	42,0	101					

Maltsters Cisterns.

Depth.	Length.	Brdth.	Area.
36.0	114	58.5	3.10
{ 10	Diam.	36.2	.48
{ 10	Diam.	33.0	.40
{ 10	Diam.	29.6	.32
30			

The above specimen was all collected from the operations in chapters ix. xi. and xiii.

CHAP. XIV.

Tables of Rates, &c.

THIS chapter contains the tables of rates and measures of all exciseable commodities, very useful for the moneying of the goods, and making out of the charges in the vouchers and abstracts.

1. *A Table of Rates and Measures of exciseable Liquors in the Country.*

Species, and by what charged.	N ^o of galls. each contains of		Rates.		
	Beer	Wine	l.	s.	d.
Strong beer per barrel —	34	—	0	8	0
Small beer per ditto —	34	—	0	1	4
Sweets per ditto —	—	31.2	0	12	0
Vinegar per ditto —	34	—	0	8	9
Mum per ditto —	—	32	1	5	0
Cyder per hogsh. exc. 6s. 8d. }	—	63	0	10	8
Ditto for malt duty, 4s. od. }	—	63	0	6	0
Ditto for new duty 1766 —	—	63	0	6	8
Verjuice per hoghead —	—	63	0	0	11
Mead or metheglin per gallon —	—	1	0	0	11

2. A Table of Rates payable by Distillers for low Wines and Spirits.

Years of com- mencement.	Low wines per gallon.				Spirits per gallon.					
	From Cyder.	From Melassies.	From Foreign wine, cyder, or foreign fruit.	From Corn, malt, &c.	From Cyder.	From Melassies.	From Corn, malt, &c.	From Foreign wine, or Cyder.	From Foreign fruits.	
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	
1736	0 1 $\frac{1}{2}$	0 6	0 6	0 1	0 3	0 3	0 3	0 6	0 3	
1743	0 1 $\frac{1}{2}$	0 6	0 6	0 1	0 3	0 3	0 3	0 6	0 3	
1746	0 0	0 0	0 3	0 0 $\frac{1}{2}$	0 1 $\frac{1}{2}$	0 0	0 1 $\frac{1}{2}$	0 3	0 1 $\frac{1}{2}$	
1751	0 1	0 0	0 0	0 1 $\frac{1}{2}$	0 3	0 0	0 4 $\frac{1}{2}$	0 0	0 0	
1760	0 6	1 3	1 3	0 5	1 1 $\frac{1}{2}$	0 8	1 3	0 8	0 8	
1762	0 1	0 3	0 3	0 1	0 2	0 2	0 3	0 2	0 2	
Tot.	1 2	2 6	2 9	0 10	2 2 $\frac{3}{4}$	1 4	2 6	2 1	1 5 $\frac{1}{2}$	

3. Table of the Rates of Candles, Soap, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Candles, tallow	0	0	1
Ditto, wax	0	0	8
Sope	0	0	1 $\frac{1}{2}$
Starch	0	0	2
Hops	0	0	1

4. A Table of the Rates of Printed Linens, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Printed silks, half yard wide, per yard	0	1	0
Silk handkerchiefs, yard square	0	0	4
Calicoes, per yard	0	0	6
Linen stuffs, per yard	0	0	3

5. *A Table of Rates of Glafs.*

Species, and by what charged.	Rates.		
	L.	s.	d.
White glafs } per c. wt. of metal	0	9	4
Green ditto }	0	2	4

6. *A Table of the Rates of Hides, &c.*

How dressed.	Species, and how charged.	Rates.		
		L.	s.	d.
Tanned.	Sheep and Lamb } Butts and backs } Hides } per pound - -	0	0	1½
	Calves and kipps }			
	Hogs and dogs }			
	Roans per pound - - - -	0	0	2
	Goats tanned with shomack per ditto	0	0	4
	Skins and pieces ad valorem per cent.	30	0	0
Tawed.	Sheep and lamb per pound - -	0	0	1¼
	Calves and kipps per ditto - -	0	0	1½
	Buck and doe per ditto - - -	0	0	6
	Calves and flink without hair } per dozen	0	1	0
	Dog and kid }			
	Beaver and goat per ditto - -	0	2	0
	Calves flink with hair per ditto -	0	3	0
	Horse hides per hide - - - -	0	1	6
	All other hides per ditto - - -	0	3	0
Dressed in oil.	Skins and pieces, &c. ad valorem per cent.	30	0	0
	Sheep and lamb per pound - -	0	0	3
	Hides, deer } per ditto - -	0	0	6
	Goat and beaver }			
	Calves skins per ditto - - -	0	0	8
	Skins and pieces, &c. ad valorem { per ditto per cent.	15	0	0
	Vellum per dozen - - - -	0	3	0
	Parchment per ditto - - - -	0	1	6

7. *Table*

All o
charged
Note

7. Tables of Equal Parts at 15l. and 30l. per Cent.

At 15l. per cent.								At 30l. per cent.							
Value	Duty	Value	Duty	Value	Duty	Value	Duty	Value	Duty	Value	Duty	Value	Duty	Value	Duty
s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
0	5	0	$\frac{3}{4}$	10	5	1	$6\frac{3}{4}$	0	$2\frac{1}{2}$	0	$\frac{3}{4}$	5	$2\frac{1}{2}$	1	$6\frac{3}{4}$
0	10	0	$1\frac{1}{2}$	10	10	1	$7\frac{1}{2}$	0	5	0	$1\frac{1}{2}$	5	5	1	$7\frac{1}{2}$
1	3	0	$2\frac{1}{4}$	11	3	1	$8\frac{1}{4}$	0	$7\frac{1}{2}$	0	$2\frac{1}{4}$	5	$7\frac{1}{2}$	1	$8\frac{1}{4}$
1	8	0	3	11	8	1	9	0	10	0	3	5	10	1	$9\frac{1}{2}$
2	1	0	$3\frac{3}{4}$	12	1	1	$9\frac{3}{4}$	1	$0\frac{1}{2}$	0	$3\frac{3}{4}$	6	$\frac{1}{2}$	1	$9\frac{3}{4}$
2	6	0	$4\frac{1}{2}$	12	6	1	$10\frac{1}{2}$	1	3	0	$4\frac{1}{2}$	6	3	1	$10\frac{1}{2}$
2	11	0	$5\frac{1}{4}$	12	11	1	$11\frac{1}{4}$	1	$5\frac{1}{2}$	0	$5\frac{1}{4}$	6	$5\frac{1}{2}$	1	$11\frac{1}{4}$
3	4	0	6	13	4	2	0	1	8	0	6	6	8	2	0
3	9	0	$6\frac{1}{4}$	13	9	2	$\frac{3}{4}$	1	$10\frac{1}{2}$	0	$6\frac{1}{4}$	6	$10\frac{1}{2}$	2	$\frac{3}{4}$
4	2	0	$7\frac{1}{2}$	14	2	2	$1\frac{1}{2}$	2	1	0	$7\frac{1}{2}$	7	1	2	$1\frac{1}{2}$
4	7	0	$8\frac{1}{4}$	14	7	2	$2\frac{1}{4}$	2	$3\frac{1}{2}$	0	$8\frac{1}{4}$	7	$3\frac{1}{2}$	2	$2\frac{1}{4}$
5	5	0	9	15	5	2	3	2	6	0	9	7	6	2	3
5	5	0	$10\frac{1}{4}$	15	5	2	$3\frac{3}{4}$	2	$8\frac{1}{2}$	0	$9\frac{1}{4}$	7	$8\frac{1}{2}$	2	$3\frac{3}{4}$
5	10	0	$10\frac{1}{2}$	15	10	2	$4\frac{1}{2}$	2	11	0	$10\frac{1}{2}$	7	11	2	$4\frac{1}{2}$
6	3	0	$11\frac{1}{4}$	16	3	2	$5\frac{1}{4}$	3	$1\frac{1}{2}$	0	$11\frac{1}{4}$	8	$1\frac{1}{2}$	3	$5\frac{1}{4}$
6	8	1	0	16	8	2	6	3	4	1	0	8	4	2	6
7	1	1	$\frac{3}{4}$	17	1	2	$6\frac{3}{4}$	3	$6\frac{1}{2}$	1	$\frac{3}{4}$	8	$6\frac{1}{2}$	2	6
7	6	1	$1\frac{1}{2}$	17	6	2	$7\frac{1}{2}$	3	9	1	$1\frac{1}{2}$	8	9	2	$7\frac{1}{2}$
7	11	1	$2\frac{1}{4}$	17	11	2	$8\frac{1}{4}$	3	$11\frac{1}{2}$	1	$2\frac{1}{4}$	8	$11\frac{1}{2}$	2	$8\frac{1}{4}$
8	4	1	3	18	4	2	9	4	2	1	5	9	2	2	9
8	9	1	$3\frac{3}{4}$	18	9	2	$9\frac{3}{4}$	4	$4\frac{1}{2}$	1	$3\frac{3}{4}$	9	$4\frac{1}{2}$	2	$9\frac{3}{4}$
9	2	1	$4\frac{1}{2}$	19	2	2	$10\frac{1}{2}$	4	7	1	$4\frac{1}{2}$	9	7	2	$10\frac{1}{2}$
9	7	1	$5\frac{1}{4}$	19	7	2	$11\frac{1}{4}$	4	$9\frac{1}{2}$	1	$5\frac{1}{4}$	9	$9\frac{1}{2}$	2	$11\frac{1}{4}$
10	0	1	6	20	0	3	10	5	0	1	6	10	0	3	0

8. A Table of the Rates of Paper, &c.

Species.	By what charged.	Rates.		
		L.	s.	d.
Demy fine	per ream	0	2	3
Ditto second	per ditto	0	1	6
Crown fine	per ditto	0	1	6
Ditto second	per ditto	0	1	$1\frac{1}{2}$
Fool's-cap fine	per ditto	0	1	6
Ditto second	per ditto	0	1	$1\frac{1}{2}$
Fine pot	per ditto	0	1	6
Ditto second	per ditto	0	0	9
Brown large cap	per ditto	0	0	9
Ditto small ordinary	per ditto	0	0	6
Whited brown	per bundle	0	0	9
Paste boards, mill boards, and scale boards	per c. wt.	0	4	6
Printed, painted, or stained paper for hangings	per yard square.	0	0	$1\frac{1}{2}$

All other papers, white or brown, of whatsoever kind or colour, are charged, ad valorem, at 18l. per cent.

Note, a bundle of paper contains two reams.

9. A Table of Equal Parts at 18l. per Cent.

Value.						Duty.					
l.	s.	d.	s.	d.	100	l.	l.	s.	d.	100	pts.
		$\frac{1}{4}$	—	—	.18	4		14	$4\frac{3}{4}$	.20	
		$\frac{1}{2}$	—	—	.36	5		18	—	—	
		$\frac{3}{4}$	—	—	.54	6	1	1	7	.80	
	1	—	—	—	.72	7	1	5	$2\frac{1}{4}$	.60	
	2	—	$1\frac{1}{4}$	—	.44	8	1	8	$9\frac{1}{2}$	.40	
	3	—	$1\frac{1}{2}$	—	.16	9	1	12	$4\frac{3}{4}$	.20	
	4	—	$1\frac{3}{4}$	—	.88	10	1	16	—	—	
	5	—	$2\frac{1}{4}$	—	.60	11	1	19	7	.80	
	6	—	—	1	.32	12	2	3	$2\frac{1}{4}$	.60	
	7	—	—	$1\frac{1}{4}$	.04	13	2	6	$9\frac{1}{2}$	.40	
	8	—	—	$1\frac{1}{4}$	.76	14	2	10	$4\frac{3}{4}$	.20	
	9	—	—	$1\frac{3}{4}$	.48	15	2	14	—	dc.	
	10	—	—	$1\frac{3}{4}$	.20	16	2	17	7	.8	
	11	—	—	$1\frac{3}{4}$	.92	17	3	1	$2\frac{1}{4}$	.6	
1	—	—	—	2	.64	18	3	4	$9\frac{1}{2}$	.4	
2	—	—	—	4	.28	19	3	8	$4\frac{3}{4}$	.2	
3	—	—	—	6	.92	20	3	12	—	—	
4	—	—	—	8	.56	21	3	15	7	.8	
5	—	—	—	10	.20	22	3	19	$2\frac{1}{4}$	.6	
6	—	—	1	$3\frac{3}{4}$	.84	23	4	2	$9\frac{1}{2}$	.4	
7	—	—	1	3	.48	24	4	6	$4\frac{3}{4}$	.2	
8	—	—	1	$5\frac{1}{4}$	.12	25	4	10	—	—	
9	—	—	1	$7\frac{1}{4}$	.76	26	4	13	7	.8	
10	—	—	1	9	.40	27	4	17	$2\frac{1}{4}$	.6	
11	—	—	1	$11\frac{1}{2}$	.04	28	5	—	$9\frac{1}{2}$	.4	
12	—	—	2	$1\frac{3}{4}$	.68	29	5	4	$4\frac{3}{4}$	.2	
13	—	—	2	4	.32	30	5	8	—	—	
14	—	—	2	6	.96	31	5	11	7	.8	
15	—	—	2	$8\frac{1}{4}$	.60	32	5	15	$2\frac{1}{4}$	.6	
16	—	—	2	10	.24	33	5	18	$9\frac{1}{2}$	.4	
17	—	—	3	$1\frac{1}{2}$	.88	34	6	2	$4\frac{3}{4}$	.2	
18	—	—	3	2	.52	35	6	6	—	—	
19	—	—	3	5	.16	36	6	9	7	.8	
1	—	—	3	7	.80	37	6	13	$2\frac{1}{4}$	.6	
2	—	—	7	$2\frac{1}{4}$	.60	38	6	16	$9\frac{1}{2}$	.4	
3	—	—	10	9	.40	39	7	—	$4\frac{3}{4}$	.2	

ad Valorem, for moneying of Paper.

V. Duty.					Val. Duty.				
l.	l.	s.	d.	10 pts	l.	l.	s.	d.	10 pts
40	7	4	—	—	76	13	13	7	.8
41	7	7	7	.8	77	13	17	2 $\frac{1}{4}$	.6
42	7	11	2 $\frac{1}{4}$	.6	78	14	—	9 $\frac{1}{2}$	.4
43	7	14	9 $\frac{1}{2}$	.4	79	14	4	4 $\frac{3}{4}$	.2
44	7	18	4 $\frac{3}{4}$	.2	80	14	8	—	—
45	8	2	—	—	81	14	11	7	.8
46	8	5	7	.8	82	14	15	2 $\frac{1}{4}$	.6
47	8	9	2 $\frac{1}{4}$	.6	83	14	18	9 $\frac{1}{2}$	.4
48	8	12	9 $\frac{1}{2}$	.4	84	15	2	4 $\frac{3}{4}$	.2
49	8	16	4 $\frac{3}{4}$	.2	85	15	6	—	—
50	9	—	—	—	86	15	9	7	.8
51	9	3	7	.8	87	15	13	2 $\frac{1}{4}$	.6
52	9	7	2 $\frac{1}{4}$	.6	88	15	16	9 $\frac{1}{2}$	.4
53	9	10	9 $\frac{1}{2}$	.4	89	16	—	4 $\frac{3}{4}$	.2
54	9	14	4 $\frac{3}{4}$	.2	90	16	4	—	—
55	10	18	—	—	91	16	7	—	.0
56	10	1	7	.8	92	16	11	2 $\frac{1}{4}$	.6
57	10	5	2 $\frac{1}{4}$	.6	93	16	14	9 $\frac{1}{2}$	.4
58	10	8	9 $\frac{1}{2}$	.4	94	16	18	4 $\frac{3}{4}$	.2
59	10	12	4 $\frac{3}{4}$	.2	95	17	2	—	—
60	10	16	—	—	96	17	5	7	.8
61	10	19	7	.8	97	17	9	2 $\frac{1}{4}$	.6
62	11	3	2 $\frac{1}{4}$	.6	98	17	12	9 $\frac{1}{2}$	.4
63	11	6	9 $\frac{1}{2}$	.4	99	17	16	4 $\frac{3}{4}$	.2
64	11	10	4 $\frac{3}{4}$	.2	100	18	—	—	—
65	11	14	—	—	200	36	—	—	—
66	11	17	7	.8	300	54	—	—	—
67	12	1	2 $\frac{1}{4}$	.6	400	72	—	—	—
68	12	4	9 $\frac{1}{2}$	.4	500	90	—	—	—
69	12	8	4 $\frac{3}{4}$	.2	600	108	—	—	—
70	12	12	—	—	700	126	—	—	—
71	12	15	7	.8	800	144	—	—	—
72	12	19	2 $\frac{1}{4}$	.6	900	162	—	—	—
73	13	2	9 $\frac{1}{2}$	.4	1000	180	—	—	—
74	13	6	4 $\frac{3}{4}$	.2	2000	360	—	—	—
75	13	10	—	—	3000	540	—	—	—

Names.

10. A Table of Wine Measure.

Names.	Cubic Inches.	Avoirdupois weight.			Quarts.	Gallons.	Rundlets.	Barrels.	Tierces.	Hogheads.	Puncheons.	Butts.	Tons.
		lb.	oz.	dr.									
Ton	58212	2042	9	4	16	1008	14	8	6	4	3	2	1
Butt	29106	1021	4	10	8	504	7	4	3	2	1½	1	
Puncheon	19404	680	13	12	672	336	4½	2½	2	1½	1		
Hoghead	14553	510	10	5	504	252	3½	2	1½	1			
Tierce	9702	340	6	14	336	168	2½	1½	1				
Barrel	7276.5	251	6	10	252	126	2	1½					
Rundlet	4158	145	14	6	144	72	1						
Gallon	231	8	1	1	8	4							
Quart	57.75	2		6¾	2	1							
Pint	28.875	1		3¾	1								

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are

11. *A Table of London Beer Measure, 36 Gallons to a Barrel.*

Names.	Cubic inches.	Avoirdupoise wt. of London beer.			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	10152	387	0	0	288	144	36	4	2	1
Kilderkin	5076	193	8	0	144	72	18	2	1	
Firkin	538	96	12	0	72	36	9	1		
Gallon	282	10	12	0	8	4	1			
Quart	70.5	2	11	0	2	1				
Pint	35.25	1	5	8	1					

12. *A Table of London Ale Measure, 32 Gallons in a Barrel.*

Names.	Cubic inches.	Avoirdupoise wt. of College ale.			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	9024.	331	6	15	256	128	32	4	2	1
Kilderkin	4512	165	11	7	128	64	16	2	1	
Firkin	2256	82	13	11½	64	32	8	1		
Gallon	282	10	5	14	8	4	1			
Quart	70.5	2	9	6½	2	1				
Pint	35.25	1	4	11	1					

13. *A Table of Beer and Ale Measure, for the Country, 34 Gallons in a Barrel.*

Names.	Cubic inches.	Avoirdupoise wt. of clear water.			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	9588	340	12	8½	272	136	34	4	2	1
Kilderkin	4794	73	6	4	136	68	17	2	1	
Firkin	2307	86	11	2	68	34	8½	1		
Gallon	282	10	3	3	8	4	1			
Quart	70.5	2	8	12½	2	1				
Pint	35.25	1	4	6¼	1					

14. *A Table of Specific Gravity, one Ounce being the Integer.*

	Names.	Ounces and		Ounces and	
		decim. parts	troy.	decim. parts	avoirdupoise
One Cubic inch of	Fine gold is — —	10.359273	11.365603		
	Standard ditto — —	9.962625	10.930422		
	Quicksilver — —	7.384411	8.101753		
	Lead — — —	5.984010	6.553885		
	Fine silver — —	5.850035	6.418324		
	Standard ditto — —	5.556769	6.096596		
	Rose copper — —	4.747121	5.208369		
	Plate brass — —	4.404273	4.832116		
	Cast brass — —	4.272409	4.630300		
	Steel — — —	4.142127	4.544505		
	Common iron — —	4.031361	4.422979		
	Block tin — —	3.861519	4.236638		
	Fine marble — —	1.429411	1.568859		
	Common glass — —	1.360841	1.493037		
	Alabaster — — —	0.988456	1.084477		
	Dry ivory — — —	0.962083	1.055542		
	London brewed ale —	0.562786	0.609929		
	Beer vinegar — —	0.545392	0.591070		
	Sack — — —	0.544864	0.590506		
	Dry box wood — —	0.543282	0.596057		
	Milk — — —	0.543809	0.589362		
	Urine — — —	0.543282	0.588791		
	College plain ale —	0.542227	0.587648		
	Salt water — —	0.542742	0.594894		
	Pump ditto — — —	0.527458	0.578697		
	Claret — — —	0.523766	0.574646		
	Linseed oil — — —	0.491591	0.539345		
	Proof spirits of brandy	0.489268	0.536796		
	Sound dry oak — —	0.489008	0.536569		
	Oil, olive — —	0.481569	0.528350		
	Oil of turpentine —	0.383680	0.415820		
	Half pint of mustard seed	5.543518	6.082031		

15. *A Table of the Avoirdupoise Weight of a Cubic Foot of several Bodies in Pounds and Decimal Parts.*

Names.	Lb.	lb.	oz	dr
Gravel	109.3125	109	5	0
Common sand	85.25	85	4	0
Newcastle coal	67.75	67	12	0
Pump water	62.734	62	12	6
Wood ashes	58.312	58	5	0
Bay salt	54.062	54	1	0
White pease	50.5	50	8	0
Field beans	50.5	50	8	0
Wheat of the best sort	48.5	48	8	0
White sea salt	43.75	43	12	0
Barley	41.125	41	2	0
Wheaten meal unfifted	31.	31	0	0
Malt 2 months old	30.25	30	4	0
White oats	29.5	29	8	0
Rye meal unfifted	28.	28	0	0

The proportion of the standard avoirdupoise weight to the standard troy weight has been found, by nice experiments made for that purpose, to be as follows, viz. that 15lb. of avoirdupoise is equal to 18lb. 2 oz. and 15 pwt. troy; so that 140 ounces of avoirdupoise is equal to 128.75 ounces of troy; which in the least terms in whole numbers is as below:

144lb. of avoirdupoise = 175 lb. of troy.

192 oz. of ditto — = 175 oz. of ditto.

By these proportions any number of pounds or ounces of avoirdupoise may be reduced to troy, and contra any number of pounds or ounces of troy may be reduced to avoirdupoise: As for example,

In 28 lb. of avoirdupoise, how many pounds of troy?

The proportion both by pen and rule is,

on A. on B. on A. on B.

As 144 : 175 :: 28 : 34.0278 = troy wt.

Avoir. lb. Troy lb. Av. lb. Troy lb.

Another

Another example.

In 16 oz. of avoirdupoise, how many ounces of troy?

PROPORTION.

on A.	on B.	on A.	on B.	
As 192 :	175 :	:	16 :	14.58 = answer.
Avoir. oz.	Troy oz.	Av. oz.	Troy oz.	

And in this manner may the avoirdupoise pounds of any of the bodies mentioned in the last table be reduced to troy weight.

And on the contrary, by transposing the first and second terms any number of pounds and ounces may be reduced from troy to avoirdupoise weight.

The chief use of the tables of specific gravity is to try weights and measures; for by knowing how many inches are contained in a solid body, we can find the weight of that body; or by having the weight of that body, we can find how many solid inches it contains.

For example: What is the avoirdupoise weight of Winchester pint of pump water?

The proportion is, as unity is to the tabular number so are the solid inches contained in that body to its weight.

on B.	on A.	on B.	on A.	lb. oz. d.
As 1 :	.578697 :	:	35.25 :	20.399 = 1 : 4 :
Unity.	Tab. numb.	Sol. inc.	Weight.	

EXAMPLE II.

Suppose a certain quantity of water weighed 14 oz. 6.38 dr. in avoirdupoise weight, how many cubic inches are contained in that water?

This is the reverse of the former, therefore the proportion is,

on A.	on B.	on A.	on B.	
As .578697 :	1 :	:	20.399 :	35.25 = answer.
Tab. numb.	Unity.	Weight.	Solid inches.	

EXA

EXAMPLE III.

How much will a bushel of wheat of the best sort weigh in avoirdupoise weight?

PROPORTION.

on B.	on A.	on B.	on A.	lb.	oz.	dr.
As 1728	: 48.5	: : 2150.42	: 60.356	= 60	: 5	: 11
Sol. incs.	Wt. of	Sol. inch.	Weight of			
in a foot.	foot.	in a bush.	a bushel.			

In like manner may the weight of any other body mentioned in the table of specific gravity be found; or, on the contrary, by knowing the weight, you may find the solidity of that body.

CHAP. XV.

Of moneying the charges.

IN this chapter is shewn the proper method of moneying the charges, or finding the amount of any quantity of goods when the books are added up, and also the method of reducing one sort of goods to its equivalent value in that of another kind; in which are contained several useful and necessary tables.

ART. I.

A short way to money goods at 1d. $\frac{1}{4}$ per pound.

RULE.

Cut off the right hand figure, which count so many pence and farthings, and those on the left hand will be so many shillings and halfpence.

K

EXAM-

EXAMPLE.

Query the duty of 769lb. of sheep skins at 1d. $\frac{1}{4}$ per pound?

OPERATION.

76 | 9 = 76 shillings, 76 halfpence, and nine times five farthings.

	l.	s.	d.
76 shillings	=	3	16 0
76 halfpence	=	0	3 2
9 times 1d. $\frac{1}{4}$	=	0	0 11 $\frac{1}{4}$

4 0 $1\frac{1}{4}$ = duty required.

ART. II.

To money goods at the rate of 30l. per cent.

RULE.

Divide the value of the goods by 5, and to the quotient add its half, whose sum will be the duty required.

EXAMPLE.

Suppose the value to be 6l. 10s. 10d. query what the duty of the same will amount to?

OPERATION.

	l.	s.	d.
5)6 10 10			
	1	6	2
one half add	0	13	1

1 19 3 = the duty required.

Or the same may be found by multiplying the value of the goods by .3.

ART. III.

To money goods at the rate of 15l. per cent.

RULE.

Divide the given value of the goods by 5, and from the quotient subtract one fourth part, and the remainder will be the duty required.

EXAMPLE.

Let the value be 5l. 8s. 4d. to find the duty.

OPERATION.

	l.	s.	d.
	5	8	4
	1	1	8
Subtract $\frac{1}{4} =$	0	5	5
	0	16	3

= duty required.

Or the same may be found by multiplying the value of the goods by .15

ART. IV.

To money goods at the rate of 18l. per cent.

RULE.

Divide the value of the goods by 5, and from that quotient subtract half of its one fifth: the remainder will be the duty required.

EXAMPLE.

Let the value of the goods be 53l. to find the duty.

K 2

OPE-

Of moneying the Charges.

OPERATION.

	l.	s.	d.
	53	0	0
$\frac{1}{3}$ of it =	10	12	0
$\frac{1}{3}$ of it =	2	2	$4\frac{2}{3}$
$\frac{1}{2}$ of the $\frac{1}{3}$ =	1	1	$2\frac{2}{3}$ substract
	9	10	$9\frac{1}{3}$ = duty required.

Or the same may be found by multiplying the value of the goods by .18 : thus,

53
.18
424
53
9.54 equal to 9l. 10s. $9\frac{1}{2}$ d. $1\frac{2}{3}$
20
10.80
12
9.6
4
2.4

ART. V.

To find the duty of any number of barrels of victuallers
~~X beer at 8s. per barrel.~~
 X beer at $\frac{10}{10}$ per barrel.

THIS may be performed several ways: but the readiest and quickest method of doing it is by multiplying the given number of barrels by $\frac{4}{5}$, and the product will be pounds, all except the units figure of the product, which will be so many two shillings, to which add the money for the firkins, if any, and that sum will be equal to the duty of the whole.

Or it may be found by the cash table in pages 206 and 207.

EXAMPLE.

What is the duty of $325\frac{1}{2}$ barrels of victuallers X beer at 8s. per barrel?

OPERATION.

By practice.
 $325 =$ given num.
 4 of barrels.

 130.0
 4 for $\frac{1}{2}$ a barrel.

 130.4

By cash table.

Bar.	l.	s.
$300 =$	120	0
$20 =$	8	0
$5 =$	2	0
$\frac{1}{2} =$	0	4
$325\frac{1}{2} =$	130	4

ART. VI.

To find the duty of any number of barrels of victuallers
 small beer at 1s. 4d. per barrel.

RULE.

TO the given number of barrels add one third of that sum, and, if there be any quarters, add proportionably for them: the sum of the whole will be equal to the duty in shillings and pence, which reduce

K 3

into

into pounds, and it is done. Or it may be found by the cash table in pages 206 and 207.

EXAMPLE.

What will the duty of $295\frac{1}{2}$ barrels of victuallers finall beer amount to at 1s. 4d. per barrel?

OPERATION.

By practice.	By the cash tables.
3)295 given number of barrels.	Bar. l. s. d.
98:4 $\frac{1}{3}$ of given number.	200 = 13 6 8
0:8 $\frac{1}{2}$ barrel.	90 = 6 0 0
	5 = 0 6 8
2 0)39 4:0 shillings.	$\frac{1}{2}$ = 0 0 8
£.19:14:0 = answer	$295\frac{1}{2} = 19 \ 14 \ 0$

And in this manner may the amount of the duty of any other sort of goods be found (if there be no fractional parts in the price of the integer), whether they be pounds, yards, &c. either by practice, or by the cash tables in pages 206 and 207; which is so easy that it needs no more examples.

A R T. VII.

To find the duty of any number of barrels of common brewers X beer at 8s. per barrel, with the allowance of $2\frac{1}{2}$ in every 23 barrels.

BY reason of this allowance of $2\frac{1}{2}$ in every 23 barrel there will be a fraction in the price of the integer so that the duty cannot be found by taking the aliquot parts, as in the two last articles. But it may be found by the cash tables for common brewers X beer in page 202 or 203, or by a factor found for that purpose which factor is found in the following manner:

Th

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199

l. s.

The duty of 23 barrels at 8s. per barrel = 9 4

Allowance out of 23 barrels is $2\frac{1}{2}$, and duty = 1 0

Duty of 23 barrels of C. brewers X — = 8 4

Then the proportion for the factor is

Barr. L. Barr. Decim.

As 23 : 8.2 : 1 : .35652 = Factor.

Now having found the above factor for C. brewers X beer, if any number of barrels, and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds and decimal parts of a pound.

EXAMPLE.

What is the duty of $350\frac{1}{2}$ barrels of common brewers X beer at 8s. per barrel, with the allowance of $2\frac{1}{2}$ barrels in 23?

OPERATION.

.35652 = factor for X.

350.5 = given number of barrels.

178260

1782600

106956

l. s. d.

124.960260 = 124 19 $2\frac{1}{2}$ = the duty.

A R T. VIII.

table To find the duty of any number of common brewers ~~small~~ beer at ~~1s. 4d.~~^{2s.} per barrel, the allowance being $2\frac{1}{2}$ in every 23 barrels.

BY reason of this allowance there will be also a fraction in the price of one barrel; so there must be a factor found for common brewers small beer as well as for the strong. Or the duty may be found by the cash table for C. brewers small beer in pages 204 and 205.

The factor is found in the following manner:

			l.	s.	d.
The duty of 23 barrels at ^{2s.} 1s. 4d. per barrel	=	1	10	8	
The allowance of $2\frac{1}{2}$ barrels	— — =	0	3	4	
The duty of 23 barrels of C. brewers } small beer — — }		=	1	7	4

Then the proportion for the factor is

Barr.	L.	Barr.	Decimal of L.
As 23	: 1.366	: : 1	: .05942 = factor.

Now having found the above factor for common brewers small beer, if any number of barrels, and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds and decimal parts of a pound.

EXAMPLE.

What will the duty of $345\frac{3}{4}$ barrels of common brewers small beer amount to at 1s. 4d. per barrel?

OPERATION.

345.75 = given number of barrels.

.05942 = factor for small beer.

69150
 138300
 311175
 172875

l. s. d.

20.5444650 = 20 10 10 $\frac{1}{2}$ 7 $\frac{1}{2}$ = duty.

K 5

Cash

Cash Table for C. Brewers X Beer and Ale at 8 Shillings

No. of bars.	0			$\frac{1}{4}$			$\frac{1}{2}$			$\frac{3}{4}$		
	L.	s.	d. 23pts	L.	s.	d. 23pts	L.	s.	d. 23pts	L.	s.	d. 23pts
0	0	0	0,	0	1	9 $\frac{1}{4}$, 13	0	3	6 $\frac{3}{4}$, 3	0	5	4, 16
1	0	7	1 $\frac{1}{2}$, 6	0	8	10 $\frac{3}{4}$, 19	0	10	8 $\frac{1}{4}$, 9	0	12	5 $\frac{1}{2}$, 22
2	0	14	3, 12	0	16	0 $\frac{1}{2}$, 2	0	17	9 $\frac{3}{4}$, 15	0	19	7 $\frac{1}{4}$, 5
3	1	1	4 $\frac{1}{2}$, 18	1	3	2, 8	1	4	11 $\frac{1}{4}$, 21	1	6	8 $\frac{3}{4}$, 11
4	1	8	6 $\frac{1}{4}$, 1	1	10	3 $\frac{1}{2}$, 14	1	12	1, 4	1	13	10 $\frac{1}{4}$, 17
5	1	15	7 $\frac{3}{4}$, 7	1	17	5, 20	1	19	2 $\frac{1}{2}$, 10	2	1	0,
6	2	2	9 $\frac{1}{4}$, 13	2	4	6 $\frac{3}{4}$, 3	2	6	4, 16	2	8	1 $\frac{1}{2}$, 6
7	2	9	10 $\frac{3}{4}$, 19	2	11	8 $\frac{1}{4}$, 9	2	13	5 $\frac{1}{2}$, 21	2	15	3, 12
8	2	17	0 $\frac{1}{2}$, 2	2	18	9 $\frac{3}{4}$, 15	3	0	7 $\frac{1}{4}$, 5	3	2	4 $\frac{1}{2}$, 18
9	3	4	2, 8	3	5	11 $\frac{1}{4}$, 21	3	7	8 $\frac{3}{4}$, 11	3	9	6 $\frac{1}{4}$, 1
10	3	11	3 $\frac{1}{2}$, 14	3	13	1, 4	3	14	10 $\frac{1}{4}$, 17	3	16	7 $\frac{1}{4}$, 7
11	3	18	5, 20	4	0	2 $\frac{1}{2}$, 10	4	2	0 $\frac{1}{4}$, 4	4	3	9 $\frac{1}{4}$, 13
12	4	5	6 $\frac{3}{4}$, 3	4	7	4, 16	4	9	1 $\frac{1}{2}$, 6	4	10	10 $\frac{1}{4}$, 11
13	4	12	8 $\frac{1}{4}$, 9	4	14	5 $\frac{1}{2}$, 22	4	16	3, 12	4	18	1 $\frac{1}{2}$, 2
14	4	19	9 $\frac{3}{4}$, 15	5	1	7 $\frac{1}{4}$, 5	5	3	4 $\frac{1}{2}$, 18	5	5	2, 8
15	5	6	11 $\frac{1}{4}$, 21	5	8	8 $\frac{3}{4}$, 11	5	10	6 $\frac{1}{4}$, 1	5	12	3 $\frac{1}{2}$, 14
16	5	14	1, 4	5	15	10 $\frac{1}{4}$, 17	5	17	7 $\frac{3}{4}$, 7	5	19	5, 20
17	6	1	2 $\frac{1}{2}$, 10	6	3	0,	6	4	9 $\frac{1}{4}$, 13	6	6	6 $\frac{3}{4}$, 3
18	6	8	4, 16	6	10	1 $\frac{1}{2}$, 6	6	11	10 $\frac{3}{4}$, 19	6	13	8 $\frac{1}{4}$, 9
19	6	15	5 $\frac{1}{2}$, 22	6	17	3, 12	6	19	0 $\frac{1}{2}$, 2	7	0	9 $\frac{3}{4}$, 15
20	7	2	7 $\frac{1}{4}$, 5	7	4	4 $\frac{1}{2}$, 18	7	6	2, 8	7	7	11 $\frac{1}{4}$, 21
21	7	9	8 $\frac{3}{4}$, 11	7	11	6 $\frac{1}{4}$, 1	7	13	3 $\frac{1}{2}$, 14	7	15	1, 4
22	7	16	10 $\frac{1}{4}$, 17	7	18	7, 7	8	0	5, 20	8	2	2 $\frac{1}{2}$, 10

The use of the table for C. Brewers X beer :

1. If the number of barrels and firkins are under 23, seek in the first column above for the number of barrels, and for the firkins, if any; at the head of the table, and at the angle of meeting, you will find the duty in pounds, shillings, pence, farthings, and 23d parts of a farthing.

2. If the given number of barrels exceed 23, then seek for the nearest less number of even barrels in the next page, and set them down with their duty; then subtract this nearest less number of barrels from the number given, and seek for the duty of the remainder in the above table, which, added to the former, will be equal to the duty of the whole.

EXAMPLE.

Required the duty of 745 $\frac{1}{2}$ barrels of C. Brewers X beer.

OPE-

per Barrel, with the Allowance of $2\frac{1}{2}$ in every 23 Barrels.

No. of barrs.	L.	s.	No. of barrs.	L.	s.	No. of barrs.	L.	s.
23	8	4	897	319	16	1771	631	8
46	16	8	920	328	0	1794	639	12
69	24	12	943	336	4	1817	647	16
92	32	16	966	344	8	1840	656	0
115	41	0	989	352	12	1863	664	4
138	49	4	1012	360	16	1886	672	8
161	57	8	1035	369	0	1909	680	12
184	65	12	1058	377	4	1912	688	16
207	73	16	1081	385	8	1955	697	0
230	82	0	1104	393	12	1978	705	4
253	90	4	1127	401	16	2001	713	8
276	98	8	1150	410	0	2024	721	12
299	106	12	1173	418	4	2047	729	16
322	114	16	1196	426	8	2070	738	0
345	123	0	1219	434	12	2093	746	4
368	131	4	1242	442	16	2116	754	8
391	139	8	1265	451	0	2139	762	12
414	147	12	1288	459	4	2162	770	16
437	155	16	1311	467	8	2185	779	0
460	164	0	1334	475	12	2208	787	4
483	172	4	1357	483	16	2231	795	8
506	180	8	1380	492	0	2254	803	12
529	188	12	1403	500	4	2277	811	16
552	197	16	1426	508	8	2300	820	0
575	205	0	1449	516	12	2323	828	4
598	213	4	1472	524	16	2346	836	8
621	221	8	1495	533	0	2369	844	12
644	229	12	1518	541	4	2392	852	16
667	238	16	1541	549	8	2415	861	0
690	246	0	1564	557	12	2438	869	4
713	254	4	1587	565	16	2461	877	8
736	262	8	1610	574	0	2484	885	12
759	270	12	1633	582	4	2507	893	16
782	278	16	1656	590	8	2530	902	0
805	287	0	1679	598	12	3060	1804	0
828	295	4	1702	606	16	7590	2706	0
851	303	8	1725	615	0	10120	3608	0
874	311	12	1748	623	4			



OPERATION.

Given number of barrs. = $745\frac{1}{2}$ L. s. d.
 The nearest less No. barrs. = 736 Duty = $262\ 8\ 0$
 The remainder — = $9\frac{1}{2}$ Duty = $3\ 7\ 8\frac{3}{4}\ 11$
 The duty of the whole — — = $265\ 15\ 8\frac{3}{4}\ 11$

K 6

Cash

Cash Table for C. Brewers Small Beer 1s. 4d. per Barrel,

No. of barrels	0			$\frac{1}{4}$			$\frac{1}{2}$			$\frac{3}{4}$		
	L.	s.	d. 23pts	L.	s.	d. 23pts	L.	s.	d. 23pts	L.	s.	d. 23pts
0	0	0	0, 0	0	0	3 $\frac{1}{2}$, 6	0	0	7, 12	0	0	10 $\frac{1}{2}$, 18
1	0	1	2 $\frac{1}{4}$, 1	0	1	5 $\frac{3}{4}$, 7	0	1	9 $\frac{1}{4}$, 13	0	2	0 $\frac{3}{4}$, 19
2	0	2	4 $\frac{1}{2}$, 2	0	2	8, 8	0	2	11 $\frac{1}{2}$, 14	0	3	3, 20
3	0	3	6 $\frac{3}{4}$, 3	0	3	10 $\frac{1}{4}$, 9	0	4	1 $\frac{3}{4}$, 15	0	4	5 $\frac{1}{4}$, 21
4	0	4	9, 4	0	5	0 $\frac{1}{2}$, 10	0	5	4, 16	0	5	7 $\frac{1}{2}$, 22
5	0	5	11 $\frac{1}{4}$, 5	0	6	2 $\frac{3}{4}$, 11	0	6	6 $\frac{1}{4}$, 17	0	6	10, 1
6	0	7	1 $\frac{1}{2}$, 6	0	7	5, 12	0	7	8 $\frac{1}{2}$, 18	0	8	0 $\frac{1}{4}$, 1
7	0	8	3 $\frac{3}{4}$, 7	0	8	7 $\frac{1}{4}$, 13	0	8	10 $\frac{3}{4}$, 19	0	9	2 $\frac{1}{2}$, 2
8	0	9	6, 8	0	9	9 $\frac{1}{2}$, 14	0	10	1, 20	0	10	4 $\frac{3}{4}$, 3
9	0	10	8 $\frac{1}{4}$, 9	0	10	11 $\frac{3}{4}$, 15	0	11	3 $\frac{1}{4}$, 21	0	11	7, 4
10	0	11	10 $\frac{1}{2}$, 19	0	12	2, 16	0	12	5 $\frac{1}{2}$, 22	0	12	9 $\frac{1}{4}$, 5
11	0	13	0 $\frac{3}{4}$, 11	0	13	4 $\frac{1}{4}$, 17	0	13	8, 1	0	13	11 $\frac{1}{2}$, 6
12	0	14	3, 12	0	14	6 $\frac{1}{2}$, 18	0	14	10 $\frac{1}{4}$, 1	0	15	1 $\frac{3}{4}$, 7
13	0	15	5 $\frac{1}{4}$, 13	0	15	8 $\frac{1}{4}$, 19	0	16	0 $\frac{1}{2}$, 2	0	16	4, 8
14	0	16	7 $\frac{1}{2}$, 14	0	16	11, 20	0	17	2 $\frac{3}{4}$, 3	0	17	6 $\frac{1}{4}$, 9
15	0	17	9 $\frac{3}{4}$, 15	0	18	1 $\frac{1}{4}$, 21	0	18	5, 4	0	18	8 $\frac{1}{2}$, 10
16	0	19	0, 16	0	19	3 $\frac{1}{2}$, 22	0	19	7 $\frac{1}{4}$, 5	0	19	10 $\frac{3}{4}$, 11
17	1	0	2 $\frac{1}{4}$, 17	1	0	6, 1	1	0	9 $\frac{1}{2}$, 6	1	1	1, 12
18	1	1	4 $\frac{1}{4}$, 18	1	1	8 $\frac{1}{4}$, 1	1	1	11 $\frac{1}{4}$, 7	1	2	3 $\frac{1}{4}$, 13
19	1	2	6 $\frac{3}{4}$, 19	1	2	10 $\frac{1}{4}$, 2	1	3	2, 8	1	3	5 $\frac{1}{4}$, 14
20	1	3	9, 20	1	4	0 $\frac{3}{4}$, 3	1	4	4 $\frac{1}{4}$, 9	1	4	7 $\frac{3}{4}$, 15
21	1	4	11 $\frac{1}{4}$, 21	1	5	3, 4	1	5	6 $\frac{1}{2}$, 10	1	5	10, 16
22	1	6	1 $\frac{1}{2}$, 22	1	6	5 $\frac{1}{4}$, 5	1	6	8 $\frac{1}{4}$, 11	1	7	0 $\frac{1}{4}$, 17

The use of the cash table for C. Brewers small beer.

1. If the number of barrels and firkins are under 23, look in the first column above for the number of barrels, and for the firkins (if there be any) at the top of this side; then at the angle of meeting you will find the exact duty in pounds, shillings, and pence, farthings and 23d parts of a farthing.

2. If the given number of barrels exceed 23, then seek for the nearest less number of even barrels on the next page, and set them down with their duty; then subtract the nearest less number of barrels, &c. from the given number of barrels, &c. and with the remainder (if any) seek above for their duty, as in the first case; which add to the former, and the sum will be equal to the duty of the whole.

EXAMPLE.

What is the duty of 345 $\frac{3}{4}$ barrels of C. Brewers small beer?

OPERA-

with the Allowance of $2\frac{1}{2}$ Barrels in every 23 Barrels.

No. of barrs.	L.	s.	d.	No. of barrs.	L.	s.	d.	No. of barrs.	L.	s.	d.
23	1	7	4	736	43	14	8	1449	86	2	0
46	2	14	8	759	45	2	0	1472	87	9	4
69	4	2	0	782	46	9	4	1495	88	16	8
92	5	9	4	805	47	16	8	1518	90	4	0
115	6	16	8	828	49	4	0	1541	91	11	4
138	8	4	0	851	50	11	4	1564	92	18	8
161	9	11	4	874	51	18	8	1587	94	6	0
184	10	18	8	897	52	6	0	1610	95	13	4
207	12	6	0	920	54	13	4	1633	97	0	8
230	13	13	4	943	56	0	8	1656	98	8	0
253	15	0	8	966	57	8	0	1679	99	15	4
276	16	8	0	989	58	15	4	1702	101	2	8
299	17	15	4	1012	60	2	8	1725	102	1	0
322	19	2	8	1035	61	10	0	1748	103	17	4
345	20	10	0	1058	62	17	4	1771	105	4	8
368	21	17	4	1081	64	4	8	1794	106	12	0
391	23	4	8	1104	65	12	0	1817	107	19	4
414	24	12	0	1127	66	19	4	1840	109	6	8
437	25	19	4	1150	68	6	8	1863	110	14	0
460	27	6	8	1173	69	14	0	1886	112	1	4
483	28	14	0	1196	71	1	4	1909	113	8	8
506	30	1	4	1219	72	8	8	1932	114	16	0
529	31	8	8	1242	73	16	0	1955	116	3	4
552	32	16	0	1265	74	3	4	1978	117	10	8
575	34	3	4	1288	76	10	8	2001	118	18	0
598	35	10	8	1311	77	18	0	2024	120	5	4
621	36	18	0	1334	79	5	4	2047	121	12	8
644	38	5	4	1357	80	12	8	2070	123	0	0
667	39	12	8	1380	82	0	0	2093	124	7	4
690	41	0	0	1403	83	7	4	2116	125	14	8
713	42	7	4	1426	84	14	8				

OPERATION.

Given No. of barrs. = 345 $\frac{3}{4}$ L. s. d.
 Next less No. of ditto = 345 Duty = 20 10 0

 Remainder - - = ... $\frac{3}{4}$ Duty = 0 0 10 $\frac{1}{2}$ 18

 The whole duty -- -- -- = 20 10 10 $\frac{1}{2}$ 18

A Cash

A Cash Table for Tanners, Tawers, Chandlers,

Numb of pds. brls. bu(h) ls.	At 1d.			At 1d. $\frac{1}{4}$			At 1d. $\frac{1}{2}$			At 2d.			At 3d.			At 4d.		
	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.
1	0	0	1	0	0	$1\frac{1}{4}$	0	0	$1\frac{1}{2}$	0	0	2	0	0	3	0	0	4
2	0	0	2	0	0	$2\frac{1}{2}$	0	0	3	0	0	4	0	0	6	0	0	8
3	0	0	3	0	0	3	0	0	$4\frac{1}{2}$	0	0	6	0	0	9	0	1	0
4	0	0	4	0	0	5	0	0	6	0	0	8	0	1	0	0	1	4
5	0	0	5	0	0	$6\frac{1}{4}$	0	0	$7\frac{1}{2}$	0	0	10	0	1	3	0	1	8
6	0	0	6	0	0	$7\frac{1}{2}$	0	0	9	0	1	0	0	1	6	0	2	0
7	0	0	7	0	0	8	0	0	$10\frac{1}{2}$	0	1	2	0	1	9	0	2	4
8	0	0	8	0	0	10	0	1	0	0	1	4	0	2	0	0	2	8
9	0	0	9	0	0	$11\frac{1}{4}$	0	1	$1\frac{1}{2}$	0	1	6	0	2	3	0	3	0
10	0	0	10	0	1	0	0	1	3	0	1	8	0	2	6	0	3	4
11	0	0	11	0	1	$1\frac{1}{4}$	0	1	$4\frac{1}{2}$	0	1	10	0	2	9	0	3	8
12	0	1	0	0	1	3	0	1	6	0	2	0	0	3	0	0	4	0
13	0	1	1	0	1	$4\frac{1}{4}$	0	1	$7\frac{1}{2}$	0	2	2	0	3	3	0	4	4
14	0	1	2	0	1	5	0	1	9	0	2	4	0	3	6	0	4	8
15	0	1	3	0	1	$6\frac{1}{4}$	0	1	$10\frac{1}{2}$	0	2	6	0	3	9	0	5	0
16	0	1	4	0	1	8	0	2	0	0	2	8	0	4	0	0	5	4
17	0	1	5	0	1	$9\frac{1}{4}$	0	2	$1\frac{1}{2}$	0	2	10	0	4	3	0	5	8
18	0	1	6	0	1	$10\frac{1}{2}$	0	2	3	0	3	0	0	4	6	0	6	0
19	0	1	7	0	1	$11\frac{1}{4}$	0	2	$4\frac{1}{2}$	0	3	2	0	4	9	0	6	4
20	0	1	8	0	2	1	0	2	6	0	3	4	0	5	0	0	6	8
30	0	2	6	0	3	$1\frac{1}{2}$	0	3	9	0	5	0	0	7	6	0	10	0
40	0	3	4	0	4	2	0	5	0	0	6	8	0	10	0	0	13	4
50	0	4	2	0	5	$2\frac{1}{2}$	0	6	3	0	8	4	0	12	6	0	16	8
60	0	5	0	0	6	3	0	7	6	0	10	0	0	15	0	1	0	0
70	0	5	10	0	7	$3\frac{1}{2}$	0	8	9	0	11	8	0	17	6	1	3	4
80	0	6	8	0	8	4	0	10	0	0	13	4	1	0	0	1	6	8
90	0	7	6	0	9	$4\frac{1}{2}$	0	11	3	0	15	0	1	2	6	1	10	0
100	0	8	4	0	10	5	0	12	6	0	16	8	1	5	0	1	13	4
200	0	16	8	1	0	10	1	5	0	1	13	4	12	10	0	3	6	8
300	1	5	0	1	11	3	1	17	6	2	10	6	13	15	0	5	0	0
400	1	13	4	2	1	8	2	10	0	3	6	8	5	0	0	6	13	4
500	2	1	8	2	12	1	3	2	6	4	3	4	6	5	0	8	6	8
600	2	10	0	3	2	6	3	15	0	5	0	0	7	10	0	10	0	0
700	2	18	4	3	12	11	4	7	6	5	16	8	8	15	0	11	23	4
800	3	6	8	4	3	4	5	0	0	6	13	4	10	0	0	13	6	8
900	3	15	0	4	13	9	5	12	6	7	10	0	11	5	0	15	0	0
1000	4	3	4	5	4	2	6	5	0	8	6	8	12	10	0	16	13	4
2000	8	6	8	10	8	4	12	10	0	16	13	4	25	0	0	33	6	8
3000	12	10	0	15	12	6	18	15	0	25	0	0	37	10	0	50	0	0
4000	16	13	4	20	16	8	25	0	0	33	6	8	50	0	0	66	13	4
5000	20	16	8	26	6	10	31	5	0	41	13	4	62	10	0	83	6	8
6000	25	0	0	31	5	0	37	10	0	50	0	0	75	0	0	100	0	0
7000	29	3	2	36	9	2	42	15	0	58	6	8	87	10	0	116	13	4
8000	33	6	8	41	13	4	50	0	0	66	13	4	100	0	0	133	6	8
9000	37	10	0	46	17	6	56	5	0	75	0	0	112	10	0	150	0	0
10000	41	13	4	52	1	8	62	10	0	83	6	8	125	0	0	166	13	4

Paper-makers, Malsters, Victuallers, &c.

At 4 ¹ . 2d.	At 6d.	At 1s. 4d.	At 4s.	At 5s.	At 6s. 8d.	At 8s.
L. s. d. 10 15	L. s. d.	L. s. d.	L. s.	L. s.	L. s. d.	L. s.
0 0 4 ¹ . 2	0 0 6	0 1 4	0 4	0 5	0 6 8	0 8
0 0 9 ¹ . 4	0 1 0	0 2 8	0 8	0 10	0 13 4	0 16
0 1 2 ¹ . 6	0 1 6	0 4 0	0 12	0 15	1 0 0	1 4
0 1 7 8	0 2 0	0 5 4	0 16	1 0	1 6 8	1 12
0 2 0	0 2 6	0 6 8	1 0	1 5	1 13 4	2 0
0 2 4 ¹ . 2	0 3 0	0 8 0	1 4	1 10	2 0 0	2 8
0 2 9 ¹ . 4	0 3 6	0 9 4	1 8	1 15	2 6 8	2 16
0 3 2 ¹ . 6	0 4 0	0 10 8	1 12	2 0	2 13 0	3 4
0 3 7 8	0 4 6	0 12 0	1 16	2 5	3 0 4	3 12
0 4 0	0 5 0	0 13 4	2 0	2 10	3 6 8	4 0
0 4 4 ¹ . 2	0 5 6	0 14 8	2 4	2 15	3 13 4	4 8
0 4 9 ¹ . 4	0 6 0	0 16 0	2 8	3 0	4 0 0	4 16
0 5 2 ¹ . 6	0 6 6	0 17 4	2 12	3 5	4 6 8	5 4
0 5 7 8	0 7 0	0 18 8	2 16	3 10	4 13 4	5 12
0 6 0	0 7 6	1 0 0	3 0	3 15	5 0 0	6 0
0 6 4 ¹ . 2	0 8 0	1 1 4	3 4	4 0	5 6 8	6 8
0 6 9 ¹ . 4	0 8 6	1 2 8	3 8	4 5	5 13 4	6 16
0 7 2 ¹ . 6	0 9 0	1 4 0	3 12	4 10	6 0 0	7 4
0 7 7 8	0 9 6	1 5 4	3 16	4 15	6 6 8	7 12
0 8 0	0 10 0	1 6 8	4 0	5 0	6 13 4	8 0
0 12 0	0 15 0	2 0 0	6 0	7 10	10 0 0	12 0
0 16 0	1 0 0	2 13 4	8 0	10 0	13 6 8	16 0
1 0 0	1 5 0	3 6 8	10 0	12 10	16 13 4	20 0
1 4 0	1 10 0	4 0 0	12 0	15 0	20 0 0	24 0
1 8 0	1 15 0	4 13 4	14 0	17 10	23 6 8	28 0
1 12 0	2 0 0	5 6 8	16 0	20 0	26 13 4	32 0
1 16 0	2 5 0	6 0 0	18 0	22 10	30 0 0	36 0
2 0 0	2 10 0	6 13 4	20 0	25 0	33 6 8	40 0
4 0 0	5 0 0	13 6 8	40 0	50 0	66 13 4	80 0
6 0 0	7 10 0	20 0 0	60 0	75 0	100 0 0	120 0
8 0 0	10 0 0	26 13 4	80 0	100 0	133 6 8	160 0
10 0 0	12 10 0	33 6 8	100 0	125 0	166 13 4	200 0
12 0 0	15 0 0	40 0 0	120 0	150 0	200 0 0	240 0
14 0 0	17 10 0	46 13 4	140 0	175 0	233 6 8	280 0
16 0 0	20 0 0	53 6 8	160 0	200 0	266 13 4	320 0
18 0 0	22 10 0	60 0 0	180 0	225 0	300 0 0	360 0
20 0 0	25 0 0	66 13 4	200 0	250 0	333 6 8	400 0
40 0 0	50 0 0	133 0 8	400 0	500 0	666 13 4	800 0
60 0 0	75 0 0	200 0 0	600 0	750 0	1000 0 0	1200 0
80 0 0	100 0 0	266 13 4	800 0	1000 0	1333 6 8	1600 0
100 0 0	125 0 0	333 6 8	1000 0	1250 0	1666 13 4	2000 0
120 0 0	150 0 0	400 0 0	1200 0	1500 0	2000 0 0	2400 0
140 0 0	175 0 0	466 13 4	1400 0	1750 0	2333 6 8	2800 0
160 0 0	200 0 0	533 6 8	1600 0	2000 0	2666 13 4	3200 0
180 0 0	225 0 0	600 0 0	1800 0	2250 0	3000 0 0	3600 0
200 0 0	250 0 0	666 13 4	2000 0	2500 0	3333 6 8	4000 0

ART. IX.

To reduce the gross bushels of malt taken in the cistern or couch, and floor, to neat bushels.

THE duty on every Winchester bushel of malt is 6d. and it is supposed that barley, after it is first wetted or steeped in the cistern, and stood there its proper time, and from thence emptied into the couch, and lain there about 30 hours, rises or increases to about $\frac{2}{3}$ part more than it was before: therefore 4 bushels in every 20 are to be allowed for that increase.

But when the malt has been out of the cistern above 30 hours it is deemed to be a floor of malt, and it is supposed that a bushel of dry barley thus wetted and steeped, &c. and afterwards thrown out into the floor, and there grown according to the usual custom, will increase or rise to two bushels, or double to what it was before: therefore 10 bushels in every 20 are to be allowed for that increase.

Now in order to find proper factors to reduce each of those bushels to their equivalent value in neat bushels, observe the following method.

1. For the cistern or couch bushels.

From 20 = bushels.

Subtract 4 = $\frac{2}{3}$

Remains 16 = $\frac{4}{5}$ = .8 = factor for cistern or couch bushels.

If any number of bushels from cistern or couch be multiplied by the above factor, the product will be equal to the neat bushels at 6d. per bushel. As for example, in 100 bushels from cistern or couch how many neat bushels?

OPERATION.

100 = bushels.

.8 = factor.

80.0 = neat bushels.

2. For

2. For the floor bushels.

There being 10 bushels in every 20 to be allowed for the increase of floor bushels, therefore the floor bushel is $= \frac{10}{20} = \frac{1}{2} = .5$, a common factor for floor bushels.

If any number of floor bushels be multiplied by the factor above found, the product will be equal to the neat bushels at 6d. each bushel. As for example, in 160 bushels from the floor how many neat bushels?

O P E R A T I O N.

160 = bushels.

.5 = factor.

80.0 = neat bushels.

A R T. X.

To find factors for reducing of couch bushels to floor bushels, and on the contrary for reducing floor bushels to couch bushels, in order to find from whence the charge will arise.

R U L E.

THE factors found in the last article are to each other as unity to the required factors: therefore the proportions are as follow:

1. As .5 : .8 :: 1 : 1.6 = factor for couch Bushels.
2. As .8 : .5 :: 1 : .625 = factor for floor bushels.

Now by these factors it will be easy to find whether the charge will arise from the best of the cistern and couch, or from the floor; for if the bushels from the best of the cistern and couch multiplied by 1.6, the factor above found, produce more bushels than the floor, the charge will be from the best: but if it produces less, then the charge will be from the floor.

Or

Or the charge may be found by multiplying the floor bushels by .625; and if the product be more than the bushels from best, the charge will be from the floor, but if less, then from the best of the cistern and couch.

E X A M P L E.

Suppose the content of a floor gage of malt = 161 bushels, and the content of the best, cistern, or couch were = 100 bushels, from which will the charge arise?

The proportion by pen or rule.

on B.	on A.	on B.	on A.
As 1	: 1.6	: : 100	: 160 = bushels.
Unity.	Factor.	Couch.	Floor.

By which I find the amount of the couch, &c. is = 160 floor bushels, that is one bushel less than the number of floor bushels: therefore the charge will arise from the floor.

Or the same may be found by this proportion.

on B.	on A.	on B.	on A.
As 1	: .625	: : 160	: 100.6 = bushels.
Unity.	Factor.	Floor.	Couch.

From whence it also appears that the floor gage is the best, viz. it amounts to the most duty.

A R T. XI.

To find the duty of any number of bushels from the best of the cistern or couch.

R U L E.

THIS may be done by the cash table in pages 206 and 207; or by a factor, which is found in the following manner:

The duty of 20 bushels of malt from cistern or couch, with the allowance of 4 in 20, or the $\frac{1}{5}$ part, is 8s. which

Of finding the Charge of Floor Bushels. 211

which reduced to the decimal of a pound sterling will = .4, and then the proportion for the factor will be

Bushels. Duty. Bushels. Duty.

As 20 : .4 :: 1 : .02 = the factor.

Now if any number of bushels from cistern or couch be multiplied by the above factor, the product will be equal to the duty in pounds and decimal parts of a pound.

EXAMPLE.

How much will the duty of 2055 bushels of malt from the best of the cistern or couch amount to at 4d. 3f. 2 tenths per bushel?

OPERATION.

	By the cash table.	By the factor.
	l. s. d.	2055 = N ^o of bushels.
2000 = bushels =	40 0 0	.02 = factor.
50 = bushels =	1 0 0	—
5 = ditto =	0 2 0	41.10
—	—	20
2055 = duty =	41 2 0	l. s. d.
	2.00 =	41 2 0

ART. XII.

To find the duty of any number of bushels from the floor.

RULE.

THIS may be done by the cash table in pages 206 and 207, or by dividing the given number of bushels by 4, and the quotient will be shillings, which reduced into pounds will be equal to the duty required.

EXAMPLE.

What is the duty of 2560 bushels of malt from the floor at 3d. per bushel?

O P E R A -

OPERATION.

By the cash table.				By practice.	
	l.	s.	d.		
2000 bushels =	25	0	0	4)2560 =	N ^o of bs.
500 ditto =	6	5	0	—	
60 ditto =	0	15	0	2 0)64 0 =	shillings.
—	—	—	—	—	
2560 duty =	32	0	0	£.32 =	duty.

In like manner may the duty of any other sort of goods be found, either by the cash table or by practice; which being so easy I shall add no more examples, but proceed to the next article.

ART. XIII.

To find factors for reducing the odd gallons of one denomination to their equivalent value in that of another denomination, so as they may produce the same duty.

RULE I.

IF the odd gallons to be reduced only differ in duty, and there be the same number of gallons to the barrel, hoghead, &c. then the factor may be found by dividing the pence that one sort is charged per barrel or hoghead, &c. by the pence that the other denomination is charged at, *vice versa*, and the quotient will be the factor required.

RULE II.

When the odd gallons to be reduced not only differ in duty but also in the number of gallons to a barrel, hoghead, &c. then the factor will be found by multiplying the pence that one sort is charged with duty by the number of gallons in a barrel, hoghead, &c. of the other for a dividend, which divided by the product of the number of gallons to the barrel, hoghead, &c. and the duty of the other, *vice versa*, the quotient will be the

the factor required, as will appear by the following examples.

EXAMPLE I.

1. To find a factor to reduce strong beer at 8s. per barrel to small at 1s. 4d. per barrel,

s. d.

8 = 96 which divided by 16

the duty of a barrel of small, the quotient will be = 6, the factor required.

2. To find a factor for reducing small beer to strong, will be only the reverse; for by turning the former divisor into a dividend it will stand thus,

96) 16. 6.166 = factor required.

EXAMPLE II.

Suppose it were required to find a factor to reduce odd gallons of cyder at 10s. 8d. per hoghead, containing 63 gallons, to its equivalent value in gallons of small beer, whose barrel contains 34 gallons: by

RULE II.

s. d. d.

The duty of a hoghead of cyder 10 8 = 128

d. galls.

and 128 × 34 = 4352 = the dividend

and 16 × 63 = 1008 = the divisor

and

$\frac{4352}{1008} = 435 = \text{the factor sought.}$

And if it were required to find a factor to reduce odd gallons of small beer to cyder it would be just the reverse; for by turning the divisor above found into a dividend,

It will stand thus $\frac{1008}{4352} = 2316 = \text{factor required.}$

And in this manner were the factors in the following table found, and as many more may be found as shall at any

any time be wanted: the use of the table is so plain and obvious that it needs not much explanation; for if you would reduce any number of odd gallons of whatsoever denomination, look into the following table for the factor you want, which multiplied by the odd gallons, the product will give the thing required.

To reduce						Multiply by		
Denomi- nations.	at how much per barr. &c.	s. d.	Denomi- nations.	at how much per barr. &c.	s. d.	Factors.		
Strong beer	8	0	Small beer	1	4	6.		
Small beer	1	4	Strong beer	8	0	0.166		
Cyder	4	0	Ditto	8	0	0.27		
Ditto	4	0	Ditto	8	0	0.405		
Ditto	6	8	Ditto	8	0	0.45		
Ditto	6	8	Ditto	8	0	0.72		
Ditto	10	8	Small beer	8	0	1.62		
Ditto	4	0		1	4	2.43		
Ditto	4	0		1	4	2.7		
Ditto	6	8		1	4	4.32		
Ditto	6	8		1	4	6.75		
Ditto	10	8		1	4	0.109		
Ditto	16	8		1	4	6.56		
Vinegar	8	9		1	4	1.62		
Ditto	8	9		1	4	3.329		
Sweets	12	0		1	4	9.72		
Mum	12	0		1	4	19.94		
Ditto	25	0		1	4	0.159		
Cyder	4	0		1	4	0.254		
Ditto	4	0		1	4			

A Table of Factors.

To find any factor, let a = number gallons, a hoghead, &c. b = number gallons in a barrel, &c. c = duty of a hoghead, &c. d = duty of a barrel, &c. and x = factor required.

Then $a :: b : c : d \times x$, and $ad \times = bc$, whence $x = \frac{bc}{ad}$; the factor required.

When the number of gallons in each are the same, and the difference is only in the price, then $a = b$ and $x = \frac{c}{d}$.

Of the use of the foregoing table of factors.

In making up the accompts, only the even quarters of a barrel or hoghead are charged, and the odd gallons (if any) are carried forward to the next accompts: but it frequently happens that a trader may not have any more of that kind of goods to be charged for a long while, and perhaps not at all; therefore it will be necessary to reduce the odd gallons which remain to any other sort of goods that are still depending, or to a fourth of the least denomination, which may be done by the help of the foregoing table.

For example: Suppose it were required to reduce 4.5 gallons of X to VI, how many gallons of VI would it produce?

OPERATION.

6.0 = factor for X to VI

4.5 = gallons of X.

300
240

27.00 = gallons of VI.

This produces 27 gallons of VI, the duty of which is equal to the duty of 4.5 of X beer.

The like may be done by the odd gallons of any other denomination mentioned in the table. But the operation is best performed by the lines A and B on the rule, which is so easy, that it requires no more examples:

A R T. XIV.

To find factors for reducing ale measure to wine and corn measure; and è contra, corn to ale and wine measure.

TO do this, observe the following proportions, which are wrought by the rule of three inverse.

P R O-

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PROPORTIONS.

$$\text{As } \left\{ \begin{array}{l} 282 : 1 :: 231 : 1.220779 \\ 231 : 1 :: 282 : .819148 \\ 282 : 1 :: 2150.42 : .131137 \\ 2150.42 : 1 :: 282 : 7.625602 \\ 231 : 1 :: 2150.42 : .107422 \\ 2150.42 : 1 :: 231 : 9.309177 \end{array} \right\} \text{ factors for } \left\{ \begin{array}{l} \text{A to W} \\ \text{W to A} \\ \text{A to C} \\ \text{C to A} \\ \text{W to C} \\ \text{C to W} \end{array} \right.$$

Of the use of the foregoing factors.

If any number of gallons of ale, wine, or corn measure be multiplied by its proper factor, the product will be the number of gallons reduced to the measure desired.

EXAMPLE.

Reduce 63 gallons of wine measure to ale measure.

OPERATION.

.819148 = factor for wine to ale measure.

63 = given number of wine gallons.

$$\begin{array}{r} 2457444 \\ 4914888 \\ \hline \end{array}$$

Prod. 51.606324 = N° of gallons in ale measure.

By this we see that 51½ gallons of ale measure are nearly equal to a wine hoghead; and thus may any number of gallons of ale measure be reduced to corn or wine measure by the help of the foregoing factors.

But for the sake of whole numbers the following proportion may serve well enough for reducing ale to wine; and *contra*, wine to ale measure, viz. as 9 is to 11, so is ale to wine measure; and as 11 is to 9, so is wine to ale measure.

For example: In 51.6 gallons of ale measure how many gallons of wine measure?

Proportion by the rule.

$$\begin{array}{cccc} \text{on B.} & \text{on A.} & \text{on B.} & \text{on A.} \\ \text{As 9} & : 11 & : : 51.6 & : 63 = \text{Wine gallons.} \end{array}$$

Pro

Of finding Factors for Salaries. 217

Proportion for wine to ale measure.

As 11 : 9 :: 63 : 51.6 = ale gallons.

By this proportion the answer comes out nearly the same as that before found by the factor for wine to ale measure.

ART. XV.

To find factors for salaries, both for common and leap years, at any rate per annum.

RULE.

AS the number of days in a year is to the salary per annum, so is one day to its salary; which decimal of the salary for one day will be a proper factor to find the salary of any number of days at that rate.

EXAMPLE.

Suppose the salary to be 5*l.* per annum, what will the factors be at that rate, both for a common year containing 365 days, and a leap year containing 366 days?

1. For a common year.

Days. L. Day. Decimal of a L.

As 365 : 5 :: 1 : 0.13699 = factor for a common yr.

$$\begin{array}{r}
 \text{I} \\
 365 \overline{) 5.00(.013699} \\
 \underline{365} \\
 1350 \\
 \underline{1095} \\
 2550 \\
 \underline{2190} \\
 3600 \\
 \underline{3285} \\
 3150 \\
 \underline{3285} \\
 135
 \end{array}$$

L

2. For

2. For the leap year.

Days. L. Day. Decimal.

As 366 : 5 :: 1 : .013661 = factor for leap year.

$$\begin{array}{r}
 \text{I} \\
 \hline
 366 \overline{) 5.00(.013661} \\
 \underline{366} \\
 1340 \\
 \underline{1098} \\
 2420 \\
 \underline{2196} \\
 .2240 \\
 \underline{2196} \\
 .440 \\
 \underline{366} \\
 \hline
 \end{array}$$

Rem. — .74

Thus have I found factors both for common and leap years, at the rate of 5l. per annum, and in this manner was the following table of factors calculated.

A Table of Factors for Salaries.

Salaries per Annum.			Factors for a common year.	Factors for a leap year.	Salaries per Annum.			Factors for a common year.	Factors for a leap year.
L.	s.	d.			L.	s.	d.		
5	0	0	.013699	.013661	90	0	0	.246575	.245902
10	0	0	.027397	.027322	100	0	0	.273973	.273224
15	0	0	.041096	.040983	115	10	0	.316439	.315574
20	0	0	.054795	.054645	120	0	0	.328767	.327869
25	0	0	.068493	.068306	200	0	0	.547945	.546448
30	0	0	.082192	.081967	300	0	0	.821918	.819672
40	0	0	.109589	.109289	400	0	0	1.095890	1.092896
48	2	6	.131849	.131489	500	0	0	1.369863	1.366120
50	0	0	.136986	.136612	600	0	0	1.643836	1.639344
52	0	0	.142466	.142076	700	0	0	1.917808	1.912568
60	0	0	.164383	.163934	800	0	0	2.191781	2.185792
70	0	0	.191781	.191257	900	0	0	2.465753	2.459016
80	0	0	.219178	.218579	1000	0	0	2.739726	2.732240
86	12	6	.237329	.233948					

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A Table for finding the Number of Days from any Day in one Month to the same Day in another Month.

Month.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	7ber	8ber	9ber	10ber	Month
Jan.	365	334	306	275	245	214	184	153	122	92	61	31	Jan.
Feb.	31	365	337	306	276	245	215	184	153	123	92	62	Feb.
March.	59	28	365	334	304	273	243	212	181	151	120	90	March
April	90	59	31	365	335	304	274	243	212	182	151	121	April
May	120	89	61	30	365	334	304	273	242	212	181	151	May
June	151	120	92	61	31	365	335	304	273	243	212	182	June
July	181	150	122	91	61	30	365	334	303	273	242	212	July
August	212	181	153	122	92	61	31	365	334	304	273	243	August
7ber	243	212	184	153	123	92	62	31	365	335	304	274	7ber
8ber	273	242	214	183	153	122	92	61	30	365	334	304	8ber
9ber	304	273	245	214	184	153	123	92	61	31	365	335	9ber
10ber	334	303	275	244	214	183	153	122	91	61	30	365	10ber

Of the use of the tables of factors for salaries, and for finding the number of days.

Of the use of the two preceding tables.

1. By the table of factors for salaires the salary due for any number of days, at any rate therein mentioned, may be found both for a common or leap year; for if you take the factor of the rate, and multiply it by the number of days, the product will be equal to the salary due, in pounds and decimal parts of a pound,

2. By the table for finding the number of days, &c. you may find the number of days that you have owing

for or due; for if you look in the side column for the month of its commencement, and in the top or bottom columns for the month that it ends in, at the angle of meeting you will find the number of days from the time of its beginning to the same day of the month that it ends in; then if the time of beginning is less than the time of ending add, if more subtract the difference to or from the number found in the table, and that sum or difference will be the exact number of days*. Or the days may be found by adding the number of days of each month included between the beginning and end of the time, as in the following example.

EXAMPLE.

Suppose the salary to be 50*l.* per annum, how much would be due to a person from the 24th of June to the 14th of August in a common year?

First for the number of days.

OPERATION.

By the table.		By adding the days in each month.	
The No. under June at the side of the table and against August in the top column is	— = 61	June 6	
Time of beginning is	—	July 31	
Ending is --- June 24	} 10	Aug. 14	
Ang. 14		—	51 = N ^o of days.
Diff. subtract --- 10	} 51 = No. of days.		

* N. B. In leap year, when the month of February is included in the time sought, add one day to the number found in the table.

Then

Of finding the Amount of Salaries. 221

Then for the salary.

The factor at 50l. per annum in a }	=	,136986
common year — — — — }		
Number of days — — — —	=	51

Salary.

l. s. d.

6	19	8 $\frac{3}{4}$	equal to	6.986286
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Another example.

Suppose the salary 50l. per annum, as before, how much would be due from the 24th of June to the 30th of August following in a leap year?

First for the number of days.

OPERATION.

By the table.

The N ^o under June at the side, and against August in the top column is	— — — —	61
Time of ending Aug. 30	} 6	
Beginning June 24		
Difference add =	6	—
Number of days =	—	67

By adding the days of each month included between

June 6

July 31

Aug. 30

67	= N ^o of days.
----	---------------------------

Then for the salary.

Factor at 50l. per annum for leap year	=	,136612
Number of days — — — —	=	67

Salary.

l. s. d.

9	3	0 $\frac{3}{4}$	fere	— — — —	=	9.153004
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The decimal is reduced by inspection according to the directions in Chap. i. Art. vii.

L 3

Note,

222 Of finding the Amount of Salaries.

Note, if any of the factors in the table are reduced, they will shew the amount of a day's salary at any rate therein mentioned.

In the foregoing examples only the gross salary is found, whereas there must be 9d. in a pound deducted for tax and charity; therefore the neat salary at 50l. per ann. is but 48l. 2s. 6d. whose factor by table is ,131849 for a common year, which multiplied by 51 = 6.724299 = 6l. 14s. 5 $\frac{3}{4}$ d. the neat salary for 51 days.

But this deduction for tax and charity may be made by multiplying the gross salary by ,9625, a decimal, and the product will be the neat salary. For example, let the gross salary be 6l. how much will the neat money be, with the allowance of 9d. in the pound?

OPERATION.

,9625 = factor.

6l. = gross salary.

———— l. s. d.

5,7750 = 5 15 6 = neat salary.

But in order to save the trouble of multiplying I have calculated the following table, where any salary therein mentioned may be found almost by inspection.

A Table of Salaries for Collectors, Supervisor, Officers, &c.

No. of days	At 25l. per cent.			At 40l.			At 50l.			At 90l.			At 120l.		
	L.s.	d.	¹⁰ pts.	L.s.	d.	¹⁰ pts.	L.s.	d.	¹⁰ pts.	L.s.	d.	¹⁰ pts.	L.s.	d.	¹⁰ pts.
1	0	1	7	0	2	2	0	8	5	4	11	7	0	6	6
2	0	2	5	0	4	4	0	5	0	9	10	4	0	13	14
3	0	4	3	0	6	6	0	2	5	0	14	1	0	19	8
4	0	5	0	0	8	9	0	0	0	0	19	8	1	6	5
5	0	6	0	0	9	1	0	1	5	1	4	5	1	12	1
6	0	8	7	0	10	1	0	8	0	1	9	3	1	19	7
7	0	10	5	0	11	2	0	1	5	1	14	0	2	6	3
8	0	12	3	0	13	4	1	2	0	1	19	7	2	0	0
9	0	14	0	0	15	6	1	4	5	1	19	4	3	7	5
10	0	17	8	0	17	8	1	7	0	1	19	5	5	9	0
20	1	7	4	1	1	1	2	14	1	2	4	1	6	11	3
30	2	1	1	2	3	5	4	2	2	3	7	2	9	17	5
40	3	2	1	4	5	7	5	9	1	4	11	4	13	3	0
50	4	3	1	5	7	9	6	11	2	5	16	6	16	8	0
60	4	4	2	6	11	7	8	11	3	6	19	7	19	0	1
70	5	4	2	7	13	7	9	11	4	7	15	8	23	3	3
80	5	5	3	8	15	3	10	12	5	8	17	0	26	6	5
90	6	5	3	9	17	3	10	12	6	0	19	1	29	11	9
100	6	16	11	10	19	3	12	13	7	3	21	2	32	17	6
200	13	13	11	21	18	4	13	13	7	13	49	5	65	15	3
300	20	11	11	32	17	6	41	11	0	19	73	7	98	12	0

Diams. in ins.	.0	.1	.2	.3	.4
12	,4011	,4078	,4146	,4214	,4282
13	,4707	,4780	,4853	,4927	,5001
14	,5459	,5537	,5616	,5695	,5775
15	,6266	,6350	,6435	,6520	,6604
16	,7130	,7219	,7309	,7400	,7491
17	,8049	,8144	,8239	,8335	,8432
18	,9024	,9124	,9225	,9327	,9429
19	1,0054	1,0160	1,0267	1,0374	1,0482
20	1,1140	1,1252	1,1364	1,1477	1,1590
21	1,2282	1,2400	1,2517	1,2635	1,2754
22	1,3480	1,3602	1,3726	1,3850	1,3974
23	1,4733	1,4861	1,4990	1,5120	1,5250
24	1,6042	1,6176	1,6310	1,6445	1,6581
25	1,7406	1,7546	1,7686	1,7827	1,7968
26	1,8827	1,8972	1,9117	1,9264	1,9411
27	2,0303	2,0454	2,0605	2,0757	2,0909
28	2,1835	2,1991	2,2148	2,2305	2,2463
29	2,3422	2,3584	2,3746	2,3910	2,4073
30	2,5066	2,5233	2,5401	2,5570	2,5739
31	2,6764	2,6938	2,7111	2,7285	2,7460
32	2,8519	2,8698	2,8877	2,9057	2,9237
33	3,0330	3,0514	3,0698	3,0884	3,1069
34	3,2196	3,2385	3,2576	3,2766	3,2958
35	3,4117	3,4312	3,4508	3,4705	3,4902
36	3,6095	3,6295	3,6497	3,6699	3,6901
37	3,8128	3,8334	3,8541	3,8749	3,8957
38	4,0217	4,0428	4,0641	4,0854	4,1068
39	4,2361	4,2578	4,2796	4,3015	4,3234
40	4,4562	4,4785	4,5008	4,5233	4,5457
41	4,6818	4,7046	4,7275	4,7505	4,7735
42	4,9129	4,9363	4,9598	4,9834	5,0069
43	5,1496	5,1736	5,1977	5,2218	5,2459
44	5,3920	5,4165	5,4411	5,4657	5,4904
45	5,6398	5,6649	5,6901	5,7153	5,7405
46	5,8933	5,9189	5,9446	5,9704	5,9962
47	6,1523	6,1785	6,2048	6,2311	6,2575
48	6,4169	6,4436	6,4705	6,4973	6,5243
49	6,6870	6,7143	6,7417	6,7692	6,7966
50	6,9628	6,9906	7,0186	7,0466	7,0746
51	7,2440	7,2725	7,3010	7,3295	7,3581
52	7,5309	7,5599	7,5890	7,6181	7,6472
53	7,8233	7,8529	7,8825	7,9122	7,9419
54	8,1214	8,1515	8,1816	8,2118	8,2421
55	8,4249	8,4556	8,4863	8,5171	8,5479
56	8,7341	8,7653	8,7966	8,8279	8,8593
57	9,0485	9,0806	9,1124	9,1443	9,1762
58	9,3691	9,4014	9,4338	9,4662	9,4988
59	9,6949	9,7278	9,7608	9,7938	9,8268

Diams. in ins.	.5	.6	.7	.8	.9
12	,4352	,4422	,4492	,4563	,4635
13	,5076	,5151	,5227	,5304	,5381
14	,5856	,5937	,6018	,6100	,6183
15	,6691	,6778	,6865	,6952	,7041
16	,7582	,7674	,7767	,7861	,7954
17	,8529	,8627	,8725	,8824	,8924
18	,9532	,9635	,9739	,9843	,9948
19	1,0590	1,0699	1,0808	1,0918	1,1029
20	1,1704	1,1819	1,1934	1,2049	1,2165
21	1,2874	1,2994	1,3114	1,3236	1,3357
22	1,4099	1,4225	1,4351	1,4478	1,4605
23	1,5380	1,5511	1,5643	1,5775	1,5908
24	1,6717	1,6854	1,6991	1,7129	1,7267
25	1,8110	1,8252	1,8395	1,8538	1,8682
26	1,9558	1,9706	1,9854	2,0003	2,0153
27	2,1062	2,1215	2,1369	2,1524	2,1679
28	2,2621	2,2780	2,2940	2,3100	2,3261
29	2,4237	2,4401	2,4567	2,4732	2,4899
30	2,5908	2,6078	2,6249	2,6420	2,6592
31	2,7635	2,7811	2,7987	2,8163	2,8341
32	2,9418	2,9599	2,9781	2,9963	3,0146
33	3,1256	3,1443	3,1630	3,1818	3,2007
34	3,3150	3,3342	3,3535	3,3728	3,3923
35	3,5099	3,5297	3,5496	3,5695	3,5894
36	3,7104	3,7308	3,7512	3,7717	3,7922
37	3,9165	3,9374	3,9584	3,9794	4,0005
38	4,1282	4,1496	4,1712	4,1927	4,2144
39	4,3454	4,3674	4,3895	4,4117	4,4338
40	4,5683	4,5908	4,6135	4,6362	4,6589
41	4,7966	4,8198	4,8430	4,8662	4,8895
42	5,0306	5,0543	5,0780	5,1019	5,1257
43	5,2701	5,2944	5,3187	5,3430	5,3675
44	5,5152	5,5400	5,5649	5,5898	5,6148
45	5,7659	5,7912	5,8167	5,8421	5,8677
46	6,0221	6,0480	6,0740	6,1000	6,1261
47	6,2839	6,3104	6,3369	6,3635	6,3902
48	6,5513	6,5783	6,6054	6,6325	6,6597
49	6,8242	6,8518	6,8794	6,9072	6,9349
50	7,1027	7,1309	7,1591	7,1873	7,2157
51	7,3868	7,4155	7,4443	7,4731	7,5020
52	7,6764	7,7057	7,7350	7,7644	7,7939
53	7,9717	8,0015	8,0314	8,0613	8,0913
54	8,2724	8,3028	8,3333	8,3638	8,3943
55	8,5788	8,6097	8,6407	8,6718	8,7029
56	8,8907	8,9222	8,9538	8,9854	9,0171
57	9,2082	9,2403	9,2724	9,3046	9,3368
58	9,5313	9,5639	9,5966	9,6293	9,6621
59	9,8600	9,8931	9,9263	9,9596	9,9930

A Table of the Areas of Circles in Ale

Diam. in ins.	.0	.1	.2	.3	.4
60	10,0264	10,0598	10,0933	10,1269	10,1605
61	10,3634	10,3974	10,4314	10,4655	10,4997
62	10,7059	10,7405	10,7751	10,8098	10,8445
63	11,0541	11,0892	11,1243	11,1596	11,1949
64	11,4078	11,4434	11,4792	11,5150	11,5508
65	11,7670	11,8033	11,8396	11,8759	11,9123
66	12,1319	12,1687	12,2055	12,2424	12,2794
67	12,5023	12,5397	12,5771	12,6145	12,6520
68	12,8783	12,9162	12,9542	12,9922	13,0303
69	13,2599	13,2982	13,3368	13,3754	13,4140
70	13,6470	13,6860	13,7251	13,7642	13,8034
71	14,0397	14,0793	14,1189	14,1586	14,1983
72	14,4380	14,4781	14,5183	14,5585	14,5988
73	14,8418	14,8825	14,9232	14,9640	15,0049
74	15,2512	15,2925	15,3338	15,3751	15,4165
75	15,6662	15,7080	15,7499	15,7918	15,8337
76	16,0867	16,1291	16,1715	16,2140	16,2565
77	16,5129	16,5558	16,5988	16,6418	16,6849
78	16,9445	16,9880	17,0316	17,0751	17,1188
79	17,3818	17,4258	17,4699	17,5141	17,5583
80	17,8246	17,8692	17,9139	17,9586	18,0033
81	18,2730	18,3182	18,3634	18,4086	18,4540
82	18,7270	18,7727	18,8185	18,8643	18,9102
83	19,1866	19,2328	19,2791	19,3255	19,3719
84	19,6517	19,6985	19,7454	19,7923	19,8393
85	20,1223	20,1697	20,2172	20,2646	20,3122
86	20,5986	20,6465	20,6945	20,7426	20,7907
87	21,0804	21,1289	21,1775	21,2261	21,2747
88	21,5678	21,6169	21,6660	21,7151	21,7643
89	22,0608	22,1104	22,1600	22,2098	22,2595
90	22,5593	22,6095	22,6597	22,7100	22,7603
91	23,0634	23,1141	23,1649	23,2157	23,2666
92	23,5731	23,6244	23,6757	23,7271	23,7785
93	24,0883	24,1402	24,1920	24,2440	24,2960
94	24,6091	24,6615	24,7140	24,7665	24,8190
95	25,1355	25,1885	25,2415	25,2945	25,3476
96	25,6675	25,7210	25,7745	25,8282	25,8818
97	26,2050	26,2591	26,3132	26,3673	26,4216
98	26,7481	26,8027	26,8574	26,9121	26,9669
99	27,2968	27,3519	27,4072	27,4625	27,5178
100	27,8510	27,9067	27,9625	28,0184	28,0743
101	28,4108	28,4671	28,5234	28,5798	28,6363
102	28,9762	29,0330	29,0899	29,1469	29,2039

Gallons from 60 to 102 Inches Diameter.

Diam. in ins.	.5	.6	.7	.8	.9
60	10,1942	10,2279	10,2617	10,2955	10,3294
61	10,5339	10,5682	10,6026	10,6370	10,6714
62	10,8793	10,9141	10,9490	10,9840	11,0190
63	11,2302	11,2656	11,3011	11,3366	11,3721
64	11,5867	11,6227	11,6587	11,6947	11,7309
65	11,9468	11,9833	12,0219	12,0585	12,0952
66	12,3164	12,3532	12,3906	12,4278	12,4650
67	12,6896	12,7272	12,7649	12,8026	12,8405
68	13,0684	13,1066	13,1448	13,1831	13,2215
69	13,4527	13,4915	13,5302	13,5691	13,6080
70	13,8426	13,8819	13,9213	13,9607	14,0002
71	14,2381	14,2780	14,3179	14,3579	14,3979
72	14,6392	14,6796	14,7201	14,7606	14,8012
73	15,0458	15,0868	15,1278	15,1689	15,2100
74	15,4580	15,4995	15,5411	15,5827	15,6244
75	15,8758	15,9178	15,9600	16,0022	16,0444
76	16,2991	16,3417	16,3844	16,4272	16,4700
77	16,7280	16,7712	16,8145	16,8578	16,9011
78	17,1625	17,2062	17,2500	17,2939	17,3378
79	17,6025	17,6468	17,6912	17,7356	17,7801
80	18,0481	18,0930	18,1379	18,1829	18,2280
81	18,4993	18,5447	18,5902	18,6358	18,6814
82	18,9561	19,0021	19,0481	19,0942	19,1403
83	19,4184	19,4650	19,5115	19,5582	19,6049
84	19,8863	19,9334	19,9806	20,0278	20,0750
85	20,3597	20,4074	20,4551	20,5029	20,5507
86	20,8388	20,8870	20,9353	20,9836	21,0320
87	21,3234	21,3722	21,4210	21,4699	21,5188
88	21,8136	21,8629	21,9123	21,9617	22,0112
89	22,3093	22,3592	22,4092	22,4592	22,5092
90	22,8107	22,8611	22,9116	22,9621	23,0128
91	23,3176	23,3685	23,4196	23,4707	23,5220
92	23,8300	23,8816	23,9332	23,9848	24,0367
93	24,3480	24,4001	24,4523	24,5045	24,5568
94	24,8716	24,9243	24,9770	25,0298	25,0826
95	25,4008	25,4540	25,5073	25,5606	25,6140
96	25,9355	25,9893	26,0432	26,0971	26,1510
97	26,4759	26,5302	26,5846	26,6390	26,6935
98	27,0217	27,0766	27,1316	27,1866	27,2416
99	27,5732	27,6286	27,6841	27,7397	27,7953
100	28,1302	28,1862	28,2423	28,2984	28,3546
101	28,6928	28,7494	28,8060	28,8627	28,9194
102	29,2610	29,3181	29,3752	29,4325	29,4898

A Table of the Areas of Circles in Ale

Diams. in ins.	.0	.1	.2	.3	.4
103	29,5471	29,6045	29,6620	29,7195	29,7771
104	30,1236	30,1816	30,2396	30,2977	30,3558
105	30,7057	30,7642	30,8228	30,8814	30,9401
106	31,2934	31,3525	31,4116	31,4708	31,5300
107	31,8866	31,9462	32,0059	32,0657	32,1255
108	32,4854	32,5456	32,6058	32,6661	32,7265
109	33,0898	33,1505	33,2113	33,2722	33,3332
110	33,6997	33,7610	33,8224	33,8838	33,9452
111	34,3152	34,3771	34,4390	34,5010	34,5630
112	34,9363	34,9987	35,0612	35,1237	35,1863
113	35,5629	35,6259	35,6889	35,7520	35,8152
114	36,1952	36,2587	36,3223	36,3859	36,4496
115	36,8329	36,8970	36,9612	37,0254	37,0896
116	37,4763	37,5409	37,6056	37,6704	37,7352
117	38,1252	38,1904	38,2557	38,3210	38,3864
118	38,7797	38,8455	38,9113	38,9772	39,0431
119	39,4398	39,5061	39,5725	39,6389	39,7054
120	40,1054	40,1723	40,2392	40,3062	40,3733
121	40,7766	40,8441	40,9116	40,9791	41,0467
122	41,4534	41,5214	41,5895	41,6575	41,7257
123	42,1358	42,2043	42,2729	42,3416	42,4103
124	42,8237	42,8928	42,9619	43,0312	43,1004
125	43,5172	43,5868	43,6566	43,7263	43,7961
126	44,2162	44,2865	44,3567	44,4271	44,4974
127	44,9209	44,9916	45,0625	45,1334	45,2043
128	45,6311	45,7024	45,7738	45,8453	45,9167
129	46,3468	46,4187	46,4907	46,5627	46,6347
130	47,0682	47,1406	47,2131	47,2857	47,3583
131	47,7951	47,8681	47,9412	48,0143	48,0874
132	48,5276	48,6011	48,6747	48,7484	48,8221
133	49,2656	49,3397	49,4139	49,4880	49,5624
134	50,0093	50,0839	50,1586	50,2334	50,3083
135	50,7584	50,8336	50,9089	50,9842	51,0596
136	51,5132	51,5889	51,6648	51,7407	51,8166
137	52,2735	52,3498	52,4262	52,5027	52,5792
138	53,0394	53,1164	53,1933	53,2703	53,3474
139	53,8109	53,8884	53,9659	54,0434	54,1211
140	54,5880	54,6660	54,7440	54,8222	54,9003
141	55,3706	55,4491	55,5278	55,6064	55,6852
142	56,1587	56,2379	56,3170	56,3963	56,4756
143	57,9525	58,0322	58,1117	58,1917	58,2716
144	58,7518	58,8321	58,9124	58,9927	59,0731
145	59,5567	59,6375	59,7184	59,7993	59,8802
146	60,3672	60,4485	60,5299	60,6114	60,6929

Gallons from 102 to 146 Inches Diameter.

Dia. in ins.	.5	.6	.7	.8	.9
103	29,8347	29,8924	29,9501	30,0079	30,0657
104	30,4140	30,4722	30,5305	30,5889	30,6473
105	30,9989	31,0577	31,1165	31,1754	31,2344
106	31,5893	31,6487	31,7081	31,7675	31,8270
107	32,1853	32,2452	32,3052	32,3652	32,4252
108	32,7869	32,8474	32,9079	32,9685	33,0291
109	33,3940	33,4551	33,5161	33,5773	33,6385
110	34,0068	34,0683	34,1300	34,1917	34,2534
111	34,6251	34,6872	34,7494	34,8116	34,8739
112	35,2489	35,3116	35,3744	35,4372	35,5000
113	35,8784	35,9416	36,0049	36,0683	36,1317
114	36,5134	36,5772	36,6418	36,7049	36,7689
115	37,1530	37,2183	37,2827	37,3472	37,4117
116	37,8002	37,8650	37,9300	37,9950	38,0601
117	38,4518	38,5173	38,5828	38,6484	38,7140
118	39,1091	39,1751	39,2412	39,3073	39,3736
119	39,7719	39,8385	39,9052	39,9719	40,0386
120	40,4403	40,5075	40,5747	40,6420	40,7093
121	41,1143	41,1820	41,2498	41,3176	41,3855
122	41,7939	41,8622	41,9305	41,9989	42,0673
123	42,4790	42,5479	42,6167	42,6856	42,7546
124	43,1697	43,2391	43,3086	43,3780	43,4476
125	43,8660	43,9360	44,0059	44,0760	44,1461
126	44,5679	44,6384	44,7089	44,7795	44,8502
127	45,2753	45,3463	45,4174	45,4886	45,5598
128	45,9883	46,0603	46,1315	46,2032	46,2750
129	46,7068	46,7791	46,8512	46,9235	46,9958
130	47,4309	47,5037	47,5764	47,6493	47,7222
131	48,1606	48,2339	48,3073	48,3806	48,4541
132	48,8959	48,9697	49,0436	49,1176	49,1916
133	49,6367	49,7111	49,7856	49,8601	49,9346
134	50,3832	50,4581	50,5331	50,6082	50,6833
135	51,1351	51,2106	51,2861	51,3618	51,4374
136	51,8926	51,9687	52,0448	52,1210	52,1972
137	52,6557	52,7324	52,8090	52,8858	52,9626
138	53,4245	53,5017	53,5789	53,6562	53,7335
139	54,1987	54,2765	54,3543	54,4321	54,5100
140	54,9786	55,0569	55,1352	55,2136	55,2921
141	55,7640	55,8428	55,9217	56,0007	56,0796
142	56,5549	56,6349	56,7138	56,7933	56,8729
143	57,3515	57,4314	57,5114	57,5915	57,6716
144	58,1536	58,2341	58,3147	58,3953	58,4760
145	58,9615	59,0423	59,1235	59,2047	59,2859
146	59,7745	59,8561	59,9378	60,0196	60,1014

130 Explanation of the Tables of Ale Areas.

Of the table of ale areas.

In the first column, under the words diam. in inches, seek for the even inches of any given diameter from 12 to 146 inches, and right against it, under .0 in the second column you will find the area to the ten thousandth part of a gallon in ale measure. But if the given diameter consists of inches and tenths of an inch, then seek the even inches of the diameter in either the 1st or 7th column, and the tenths in any of the other 9 columns at the head of the table, and at the angle of meeting you will find the area as before.

EXAMPLE I.

Suppose it were required to find the ale area of a circle whose diameter was 60 inches. I look for 60 in the 1st column, and right against it in the 2d column under .0 I find $10.0264 =$ the area in ale gallons.

EXAMPLE II.

Suppose the given diameter were 75.3 inches, what is the area in ale gallons?

I find 75 in the first column, and the 3 tenths at the head of the table, then at the angle of meeting I find $15.7918 =$ the area in ale gallons required.

EXAMPLE III.

Suppose the given diameter was 78.7 inches: to find the area in ale gallons.

I find 78 in the 7th column, and the 7 tenths at the head of the table, then at the angle of meeting is 17.25, the area in ale gallons required.

And thus may the area of any other diameter within the compass of the table be found. But if you should want to know the ale area of a circle of a less or greater diameter than any contained in the table, you may find it by the following directions.

1. When the given diameter is under 12 inches, seek for the area of double that diameter, and divide it by 4, and the quotient will = the area required.

2. If

Explanation of the Tables of Ale Areas. 231

2. If the giving diameter exceeds 146, seek for the area of half that diameter in the table, and multiply it by 4, and that product will be equal to the area sought. For example in the first case: Suppose it were required to find the ale area of a circle whose diameter is 6 inches, I look in the table for the area of 12 inches, the double of 6, which I find to be = .4011, which divided by 4 = .100275 = area required.

Example in the second case.

Suppose the given diameter 160 inches, what is the area thereof in ale gallons?

I seek for 80 = half the given diameter in the table, where I find the area = 17.8246, which multiplied by 4 is = 71.2984 = to the area required.

And in this manner the area of any other diameter that exceeds the bounds of the table may be found.

Table to convert gallons into barrels, & contra.

Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
0	0	8,5	17	25,5	21	714	722,5	731	739,5
1	34	42,5	51	59,5	22	748	756,5	765	773,5
2	68	76,5	85	93,5	23	782	790,5	799	807,5
3	102	110,5	119	127,5	24	816	824,5	833	841,5
4	136	144,5	153	161,5	25	850	858,5	867	875,5
5	170	178,5	187	195,5	26	884	892,5	901	909,5
6	204	212,5	221	229,5	27	918	926,5	935	943,5
7	238	246,5	255	263,5	28	952	960,5	969	977,5
8	272	280,5	289	297,5	29	986	994,5	1003	1011,5
9	306	314,5	323	331,5	30	1020	1028,5	1037	1045,5
10	340	348,5	357	365,5	31	1054	1062,5	1071	1079,5
11	374	382,5	391	399,5	32	1088	1096,5	1105	1113,5
12	408	416,5	425	433,5	33	1122	1130,5	1139	1147,5
13	442	450,5	459	467,5	34	1156	1164,5	1173	1181,5
14	476	484,5	493	501,5	35	1190	1198,5	1207	1215,5
15	510	518,5	527	535,5	36	1224	1232,5	1241	1249,5
16	544	552,5	561	569,5	37	1258	1266,5	1275	1283,5
17	578	586,5	595	603,5	38	1292	1300,5	1309	1317,5
18	612	620,5	629	637,5	39	1326	1334,5	1343	1351,5
19	646	654,5	663	671,5	40	1360	1368,5	1377	1385,5
20	680	688,5	697	705,5	41	1394	1402,5	1411	1419,5

To

232 To convert Gallons of Ale into Barrels.

To convert any number of gallons into barrels, seek the given number of gallons, or the nearest less, in any of the columns except that under barrels, and right against it to the left hand under barrels are the barrels; and at the head of the table in the same column the quarters, if any.

E X A M P L E I.

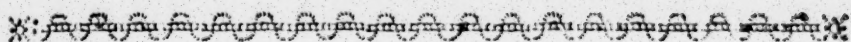
Suppose it were required to reduce 255 gallons to barrels, &c.

I find 255 in the fourth column under $\frac{1}{2}$, and right against it under barrels is 7, which added together, is 7 B. and $\frac{1}{2}$ = 255 gallons.

E X A M P L E II.

Suppose it were required to reduce 250 gallons into barrels, &c.

I find in the third column the nearest less to 250 is 246.5, which is 3.5 less than the giving number. Against it, on the left hand under barrels, is 7, and above it at the head is $\frac{1}{4}$: so the whole is 7 B. 1 F. and 3.5 G. = 250 gallons; which is so easy, that it would be needless to add any more examples.



A P P E N D I X.

ART. I.

To gage a still, and find the content therefore on every inch of its depth.

RULE.

1. **S**UPPOSE the still to be divided into three parts by the lines $c d c$ and $A o A$; then that part from the collar cc to the place where the nails join it to the body at $c c$ is supposed to be globular, and may be gaged as the middle frustum or zone of a sphere, but in practice by taking mean diameters in the middle of every 3 or 4 inches of its depth.

2. The middle part or body of the still, viz. from $c c$ to $A A$, is taken for the middle frustum of a spheroid, and as such may be gaged, but in practice by taking of mean diameters in the middle of every 6 inches of its depth.

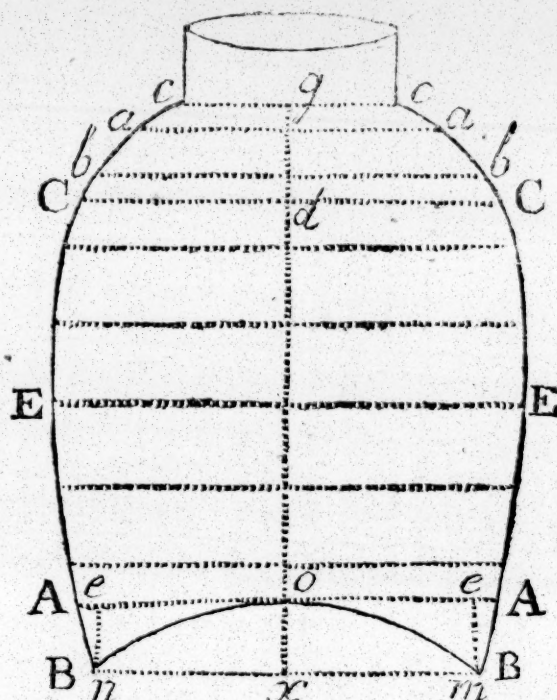
3. The lower part $A o A$, $B o B$, or that part which includes the crown, may be gaged as the like part of the copper; or in practice by pouring in water from a gallon, or some known measure, till the crown is covered, which will be when the water touches the point o .

EXAMPLE.

Admit the following still was to be gaged, and the content required at every inch of the depth in wine gallons.

A S T I L L.

A STILL.



Suppose with an instrument I found the depth of the body of the still $do = 30$ inches, and that of the globular part $gd = 7$ inches, then the whole depth from the top of the crown to the collar will be 37 inches $= go$; and suppose with a sliding cane I found the first mean diameter aa in the middle of the upper 3 inches of the globular part $= 54$ inches, and the second mean diameter bb in the middle of the next 4 inches $= 60.8$, as is set down in the second column of the following table.

Next I take mean diameters in the middle of every 6 inches of the spheroidal part AEC , CEA , and set them also down as in the second column of the table.

And suppose I found by measure that the quantity of water to cover the crown is $= 34$ gallons, which I set down in the fourth column of the table.

By

By the tables of wine areas in pages 242, &c. I find the area of each diameter, and place them against their respective diameters in the third column of the table.

Then I multiply the several areas by the respective depths to which they belong, and set the products in the fourth column, which are the contents of the separate parts; whose sum, with the quantity of liquor to cover the crown, will be equal to $606.842 =$ the whole content of the still.

Depth.	Diam.	Area.	Cont.
3	54.0	9,914	29,742
4	60.8	12,568	50,272
6	67.0	15,263	91,578
6	71.0	17,140	102,840
6	72.0	17,626	105,756
6	70.2	16,755	100,530
6	67.2	15,354	92,124
To cover Cr.			34,
37	Whole cont.		606,842

To find how much the still will hold on every inch proceed as follows:

Whole

Whole cont.	Continued.	Continued.
606.842	435.250	209.899
Sub. 9.914	Note 17.140	16.755
1 = 596.928	14 = 418.110	27 = 193.144
9.914	17.140	16.755
2 = 587.014	15 = 400.970	28 = 176.389
9.914	17.140	16.755
3 = 577.100	16 = 383.830	29 = 159.634
Note 12.568	17.140	16.755
4 = 564.532	17 = 366.690	30 = 142.879
12.568	17.140	16.755
5 = 551.964	18 = 349.550	31 = 126.124
12.568	17.140	Note 15.354
6 = 539.396	19 = 332.410	32 = 110.770
12.568	Note 17.626	15.354
7 = 526.828	20 = 314.784	33 = 95.416
Note 15.263	17.626	15.354
8 = 511.565	21 = 297.158	34 = 80.062
15.263	17.626	15.354
9 = 496.302	22 = 279.532	35 = 64.708
15.263	17.626	15.354
10 = 481.039	23 = 261.906	36 = 49.354
15.263	17.626	15.354
11 = 465.776	24 = 244.280	37 = 34.000
15.263	17.626	Crown. 34
12 = 450.513	25 = 226.654	00000
15.263	Note 16.755	
13 = 435.250	26 = 209.899	

A Table shewing the Content of the Still at every dry Inch, to the nearest even Gallon, collected from the preceding Work.

Dry ins.	Galls.	Dry ins.	Galls.	Dry ins.	Galls.
Full.	607	13	435	26	210
1	597	14	418	27	193
2	587	15	401	28	176
3	577	16	384	29	160
4	565	17	367	30	143
5	552	18	350	31	126
6	539	19	332	32	111
7	527	20	315	33	95
8	512	21	297	34	80
9	496	22	280	35	65
10	481	23	262	36	49
11	466	24	244	37	34
12	451	25	227	Cov. Cn.	34

ART. II.

To find the content of the still without taking mean diameters.

1. **F**OR the content of the globular part $c c c c$, sup-
posing it to be the middle frustum or zone of a
sphere.

RULE.

To half the sum of the squares of the top and bottom
diameters $c c$ and $c c$ add two thirds of the square of the
altitude $g d$, and that sum multiply by the altitude; this
product divide by 294.12 and the quotient will give the
content of that part in wine gallons. *Vide Simpson's*
Fluxions, page 212, 1st edit.

EXAMPLE.

Suppose the

Top diam. $c c = 50.5$	} inches {	Sq. is }	2550.25
Bot. diam. $c c = 64$			4096.
Sum of the squares			= 6646.25
Half sum			= 3323.125
Two thirds of square of alt. 7			= 32.666
Sum			= 3355.791
Multiply by the altitude			= 7
Product			= 23490.537

AT

APPENDIX.

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EXAMPLE.

Let the

Diameter A A at top of crown = 64 } areas { 13.926

Alt. of crown $ox = en = em$ — = 5 } areas { .085

Add $\frac{1}{3}$ of the area of 5 — — = .028

Sum subtract from the area of the top of the crown — — } = .113

Remainder — — — = 13.813

Multiply by $\frac{1}{2}$ the height of crown — = 2.5

69065

27626

Quantity of liquor to cover the crown = 34.5325

The body of the still is — — = 491.772

Globular part is — — = 79.867

The whole content is — — — = 606.1715

The content found in this manner is within a gallon of that found before by taking mean diameters.

A Casb

A Cash Table for Sweets at 12s. per Barrel.

Galls.	s.	d.	7 pts. of a farth.	Galls.	s.	d.	7 pts. of a farth.	Barrels	L.	s.
$\frac{1}{4}$	0	1	4	16	6	1	4	2	1	4
$\frac{1}{2}$	0	2	1	17	6	$5\frac{1}{2}$	6	3	1	16
$\frac{3}{4}$	0	3	5	18	6	$10\frac{3}{4}$	1	4	2	8
1	0	4	2	19	7	$2\frac{3}{4}$	3	5	3	0
2	0	9	4	20	7	$7\frac{1}{4}$	5	6	3	12
3	1	$1\frac{1}{4}$	6	21	8	0	0	7	4	4
4	1	$6\frac{1}{4}$	1	22	8	$4\frac{1}{2}$	2	8	4	16
5	1	$10\frac{3}{4}$	3	23	8	9	4	9	5	8
6	2	$3\frac{1}{4}$	5	24	9	$1\frac{1}{2}$	6	10	6	0
7	2	8	0	25	9	$6\frac{1}{4}$	1	20	12	0
8	3	0	2	26	9	$10\frac{3}{4}$	3	30	18	0
9	3	5	4	27	10	$3\frac{1}{4}$	5	40	24	0
10	3	$9\frac{1}{2}$	6	28	10	8	0	50	30	0
11	4	$2\frac{1}{4}$	1	29	11	$0\frac{1}{2}$	2	60	36	0
12	4	$6\frac{3}{4}$	3	30	11	5	4	70	42	0
13	4	$11\frac{1}{4}$	5	31	11	$9\frac{1}{2}$	6	80	48	0
14	5	4	0	31	12	0	0	90	54	0
15	5	$8\frac{1}{2}$	2					100	60	0

Of the use of the above Table.

1. If it be required to find the duty of any number of gallons and quarts of sweets,

Find the given number of gallons in the first or fifth column under the word gallons, then against it on the right hand is the duty in shillings, pence, and 7th parts of a farthing: if there be any quarts find them in the first column, and right against them is the duty, which added to the duty of the gallons found before, will give the duty of the number required.

2. If the given number consists of barrels, gallons, and quarts, then seek the barrels in the 9th column under the word barrels, and against them on the right hand is the duty, which added to the duty of the gallons and quarts found by the first case, will give the duty of the number required. An example or two will make this quite plain.

EXAM-

APPENDIX.

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EXAMPLE I.

What is the duty of $7\frac{1}{2}$ gallons of sweets?

	s.	d.	pt.
Against 7 in the first column is	2	8	0
Against $\frac{1}{2}$ in the same column is	0	$2\frac{1}{4}$	1

Their sum = duty required = $2 : 10\frac{1}{4} : 1$

EXAMPLE II.

What is the duty of 10 B. $5\frac{1}{2}$ gs. of sweets?

	l.	s.	d	pt.
Against 10 in the 9th col. under bar. is	6	0	0	0
Against 5 in the first col. under gall. is	0	1	$10\frac{3}{4}$	3
Against $\frac{1}{2}$ in the same column is	—	0	0	$2\frac{1}{4}$

Sum is equal to the duty required = $6 : 2 : 1 : 4$

APPENDIX.

4. A Table of the Areas of Circles in Wine

Diams. in ins.	.0	.1	.2	.3	.4
1	0,0034	0,0041	0,0049	0,0057	0,0067
2	0,0136	0,0150	0,0165	0,0180	0,0196
3	0,0306	0,0327	0,0348	0,0370	0,0393
4	0,0544	0,0572	0,0600	0,0629	0,0658
5	0,0850	0,0884	0,0919	0,0955	0,0991
6	0,1224	0,1265	0,1307	0,1349	0,1393
7	0,1666	0,1714	0,1763	0,1812	0,1862
8	0,2176	0,2231	0,2286	0,2342	0,2399
9	0,2754	0,2816	0,2878	0,2941	0,3004
10	0,3400	0,3468	0,3537	0,3607	0,3677
11	0,4114	0,4189	0,4265	0,4341	0,4419
12	0,4896	0,4978	0,5061	0,5144	0,5228
13	0,5746	0,5835	0,5924	0,6014	0,6105
14	0,6664	0,6760	0,6856	0,6953	0,7050
15	0,7650	0,7752	0,7855	0,7959	0,8063
16	0,8704	0,8813	0,8923	0,9033	0,9145
17	0,9826	0,9942	1,0059	1,0176	1,0294
18	1,1016	1,1139	1,1262	1,1386	1,1511
19	1,2274	1,2404	1,2534	1,2665	1,2796
20	1,3600	1,3736	1,3873	1,4011	1,4149
21	1,4994	1,5137	1,5281	1,5425	1,5571
22	1,6456	1,6606	1,6757	1,6908	1,7060
23	1,7986	1,8143	1,8300	1,8458	1,8617
24	1,9584	1,9748	1,9912	2,0077	2,0242
25	2,1250	2,1420	2,1591	2,1763	2,1935
26	2,2984	2,3161	2,3339	2,3517	2,3697
27	2,4786	2,4970	2,5155	2,5340	2,5526
28	2,6656	2,6846	2,7038	2,7230	2,7423
29	2,8594	2,8792	2,8990	2,9189	2,9388
30	3,0600	3,0804	3,1009	3,1215	3,1421
31	3,2674	3,2885	3,3097	3,3309	3,3523
32	3,4816	3,5034	3,5253	3,5472	3,5692
33	3,7026	3,7251	3,7476	3,7702	3,7929
34	3,9304	3,9536	3,9768	4,0001	4,0234
35	4,1650	4,1888	4,2127	4,2367	4,2607
36	4,4064	4,4309	4,4555	4,4801	4,5049
37	4,6546	4,6798	4,7051	4,7304	4,7558
38	4,9096	4,9355	4,9614	4,9874	5,0133
39	5,1714	5,1980	5,2246	5,2513	5,2780
40	5,4400	5,4672	5,4945	5,5219	5,5493
41	5,7154	5,7433	5,7713	5,7993	5,8275
42	5,9976	6,0262	6,0549	6,0836	6,1124
43	6,2866	6,3159	6,3452	6,3746	6,4041
44	6,5824	6,6124	6,6424	6,6725	6,7026
45	6,8850	6,9156	6,9463	6,9771	7,0079

APPENDIX.

243

Gallons from 1 to 46 Inches Diameter.

Diam. in ins.	5.	.6	.7	.8	.9
1	0,0077	0,0087	0,0098	0,0110	0,0123
2	0,0213	0,0230	0,0248	0,0267	0,0286
3	0,0417	0,0441	0,0465	0,0491	0,0517
4	0,0689	0,0719	0,0751	0,0783	0,0816
5	0,1029	0,1066	0,1104	0,1144	0,1184
6	0,1437	0,1481	0,1526	0,1572	0,1619
7	0,1913	0,1964	0,2016	0,2069	0,2122
8	0,2457	0,2515	0,2573	0,2633	0,2693
9	0,3069	0,3133	0,3199	0,3265	0,3332
10	0,3749	0,3820	0,3893	0,3966	0,4040
11	0,4497	0,4575	0,4654	0,4734	0,4815
12	0,5313	0,5398	0,5484	0,5571	0,5658
13	0,6197	0,6289	0,6381	0,6475	0,6569
14	0,7149	0,7247	0,7347	0,7447	0,7548
15	0,8169	0,8274	0,8381	0,8488	0,8596
16	0,9257	0,9369	0,9482	0,9596	0,9711
17	1,0413	1,0532	1,0652	1,0773	1,0894
18	1,1637	1,1763	1,1889	1,2017	1,2145
19	1,2929	1,3061	1,3195	1,3329	1,3464
20	1,4289	1,4428	1,4569	1,4710	1,4852
21	1,5717	1,5863	1,6010	1,6158	1,6307
22	1,7213	1,7366	1,7520	1,7675	1,7830
23	1,8777	1,8937	1,9097	1,9259	1,9421
24	2,0409	2,0575	2,0743	2,0911	2,1080
25	2,2109	2,2282	2,2457	2,2632	2,2808
26	2,3877	2,4057	2,4238	2,4420	2,4603
27	2,5713	2,5900	2,6088	2,6277	2,6466
28	2,7617	2,7811	2,8005	2,8201	2,8396
29	2,9589	2,9789	2,9991	3,0193	3,0396
30	3,1629	3,1836	3,2045	3,2254	3,2464
31	3,3737	3,3951	3,4166	3,4382	3,4599
32	3,5913	3,6134	3,6356	3,6579	3,6802
33	3,8157	3,8385	3,8613	3,8843	3,9073
34	4,0469	4,0703	4,0939	4,1175	4,1412
35	4,2849	4,3090	4,3333	4,3576	4,3820
36	4,5297	4,5545	4,5794	4,6044	4,6295
37	4,7813	4,8068	4,8324	4,8580	4,8838
38	5,0397	5,0659	5,0921	5,1185	5,1449
39	5,3049	5,3317	5,3587	5,3857	5,4128
40	5,5769	5,6044	5,6321	5,6598	5,6876
41	5,8557	5,8839	5,9122	5,9406	5,9691
42	6,1413	6,1702	6,1992	6,2283	6,2574
43	6,4337	6,4633	6,4929	6,5227	6,5525
44	6,7328	6,7631	6,7935	6,8239	6,8544
45	7,0389	7,0698	7,1000	7,1320	7,1632

APPENDIX.

A Table of the Areas of Circles in Wine

Diams. in ins.	.0	.1	.2	.3	.4
46	7,1944	7,2257	7,2571	7,2885	7,3201
47	7,5106	7,5426	7,5747	7,6068	7,6390
48	7,8336	7,8663	7,8990	8,9318	7,9647
49	8,1634	8,1968	8,2302	8,2637	8,2972
50	8,5000	8,5340	8,5681	8,6023	8,6365
51	8,8434	8,8781	8,9129	8,9477	8,9827
52	9,1936	9,2290	9,2645	9,3000	9,3356
53	9,5506	9,5867	9,6228	9,6590	9,6953
54	9,9144	9,9512	9,9879	10,0248	10,0618
55	10,2850	10,3224	10,3599	10,3975	10,4351
56	10,6624	10,7005	10,7387	10,7769	10,8153
57	11,0466	11,0854	11,1243	11,1632	11,2022
58	11,4376	11,4771	11,5166	11,5562	11,5959
59	11,8354	11,8756	11,9158	11,9561	11,9964
60	12,2400	12,2808	12,3217	12,3627	12,4037
61	12,6514	12,6929	12,7344	12,7761	12,8178
62	13,0696	13,1117	13,1540	13,1963	13,2388
63	13,4946	13,5374	13,5804	13,6234	13,6665
64	13,9264	13,9698	14,0136	14,0571	14,1011
65	14,3650	14,4092	14,4536	14,4976	14,5424
66	14,8104	14,8553	14,9002	14,9453	14,9904
67	15,2626	15,3081	15,3538	15,3995	15,4454
68	15,7216	15,7678	15,8142	15,8606	15,9072
69	16,1874	16,2343	16,2812	16,3284	16,3756
70	16,6600	16,7076	16,7552	16,8031	16,8508
71	17,1394	17,1877	17,2360	17,2845	17,3330
72	17,6256	17,6745	17,7236	17,7727	17,8220
73	18,1186	18,1682	18,2180	18,2678	18,3176
74	18,6184	18,6687	18,7192	18,7696	18,8202
75	19,1250	19,1760	19,2272	19,2783	19,3296
76	19,6384	19,6901	19,7418	19,7937	19,8456
77	20,1586	20,2109	20,2634	20,3159	20,3684
78	20,6856	20,7386	20,7918	20,8450	20,8983
79	21,2194	21,2731	21,3268	21,3808	21,4348
80	21,7600	21,8144	21,8688	21,9235	21,9780
81	22,3074	22,3625	22,4176	22,4729	22,5282
82	22,8616	22,9173	22,9732	23,0291	23,0852
83	23,4226	23,4790	23,5356	23,5922	23,6488
84	23,9904	24,0475	24,1048	24,1620	24,2194
85	24,5650	24,6228	24,6808	24,7387	24,7968
86	25,1464	25,2049	25,2634	25,3221	25,3808
87	25,7346	25,7937	25,8530	25,9123	25,9715
88	26,3296	26,3894	26,4494	26,5094	26,5694
89	26,9314	26,9919	27,0524	27,1132	27,1740
90	27,5400	27,6012	27,6624	27,7239	27,7853

APPENDIX.

245

Gallons from 46 to 90 Inches Diameter.

Diam. in ins.	.5	.6	.7	.8	.9
46	7,3517	7,3833	7,4150	7,4468	7,4787
47	7,6713	7,7036	7,7359	7,7685	7,8010
48	7,9976	8,0307	8,0637	8,0969	8,1301
49	8,3309	8,3645	8,3983	8,4321	8,4660
50	8,6709	8,7052	8,7397	8,7742	8,8088
51	9,0177	9,0527	9,0878	9,1230	9,1583
52	9,3713	9,4070	9,4428	9,4787	9,5146
53	9,7317	9,7681	9,8045	9,8411	9,8777
54	10,0989	10,1359	10,1731	10,2103	10,2476
55	10,4728	10,5106	10,5485	10,5864	10,6244
56	10,8537	10,8921	10,9306	10,9692	11,0079
57	11,2413	11,2804	11,3196	11,3588	11,3982
58	11,6357	11,6755	11,7153	11,7553	11,7953
59	12,0369	12,0773	12,1179	12,1585	12,1992
60	12,4449	12,4860	12,5273	12,5686	12,6100
61	12,8595	12,9016	12,9434	12,9854	13,0274
62	13,2812	13,3236	13,3663	13,4090	13,4517
63	13,7096	13,7523	13,7961	13,8394	13,8829
64	14,1448	14,1888	14,2327	14,2768	14,3208
65	14,5868	14,6314	14,6760	14,7208	14,7655
66	15,0356	15,0808	15,1262	15,1716	15,2170
67	15,4912	15,5372	15,5831	15,6292	15,6753
68	15,9536	16,0002	16,0469	16,0936	16,1405
69	16,4228	16,4700	16,5175	16,5648	16,6124
70	16,8988	16,9468	16,9948	17,0428	17,0911
71	17,3816	17,4303	17,4790	17,5278	17,5766
72	17,8712	17,9204	17,9699	18,0194	18,0689
73	18,3676	18,4176	18,4677	18,5178	18,5681
74	18,8708	18,9216	18,9723	19,0232	19,0740
75	19,3808	19,4322	19,4836	19,5352	19,5867
76	19,8976	19,9496	20,0018	20,0540	20,1062
77	20,4212	20,4740	20,5267	20,5796	20,6325
78	20,9516	21,0049	21,0585	21,1120	21,1657
79	21,4888	21,5428	21,5971	21,6512	21,7056
80	22,0328	22,0876	22,1424	22,1973	22,2523
81	22,5836	22,6392	22,6946	22,7502	22,8058
82	23,1412	23,1972	23,2535	23,3098	23,3661
83	23,7056	23,7624	23,8193	23,8762	23,9333
84	24,2768	24,3344	24,3919	24,4496	24,5072
85	24,8548	24,9130	24,9712	25,0296	25,0870
86	25,4390	25,4985	25,5574	25,6164	25,6754
87	26,0312	26,0908	26,1503	26,2100	26,2697
88	26,6296	26,6898	26,7401	26,8104	26,8709
89	27,2348	27,2956	27,3567	27,4176	27,4788
90	27,8468	27,9084	27,9700	28,0316	28,0935

APPENDIX.

A Table of the Areas of Circles in Wine

Diams. in ins.	.0	.1	.2	.3	.4
91	28,1554	28,2173	28,2792	28,3413	28,4034
92	28,7776	28,8401	28,9028	28,9655	29,0283
93	29,4066	29,4698	29,5332	29,5966	29,6601
94	30,0422	30,1063	30,1703	30,2344	30,2986
95	30,6850	30,7496	30,8143	30,8791	30,9439
96	31,3344	31,3997	31,4650	31,5305	31,5960
97	31,9906	32,0565	32,1226	32,1887	32,2549
98	32,6536	32,7202	32,7870	32,8538	32,9207
99	33,3234	33,3907	33,4581	33,5256	33,5932
100	33,9999	34,0680	34,1361	34,2043	34,2725
101	34,6834	34,7521	34,8208	34,8897	34,9586
102	35,3736	35,4429	35,5124	35,5819	35,6515
103	36,0706	36,1406	36,2108	36,2810	36,3513
104	36,7744	36,8451	36,9159	36,9868	37,0578
105	37,4850	37,5564	37,6279	37,6995	37,7711
106	38,2024	38,2745	38,3466	38,4189	38,4912
107	38,9266	38,9993	39,0722	39,1451	39,2181
108	39,6576	39,7310	39,8046	39,8782	39,9519
109	40,3954	40,4695	40,5437	40,6180	40,6924
110	41,1400	41,2148	41,2898	41,3647	41,4397
111	41,8914	41,9669	42,0424	42,1181	42,1938
112	42,6496	42,7257	42,8019	42,8783	42,9547
113	43,4146	43,4914	43,5684	43,6454	43,7225
114	44,1864	44,2639	44,3415	44,4192	44,4970
115	44,9650	45,0432	45,1215	45,1999	45,2783
116	45,7504	45,8293	45,9082	45,9873	46,0664
117	46,5426	46,6221	46,7018	46,7815	46,8613
118	47,3416	47,4218	47,5022	47,5826	47,6631
119	48,1474	48,2283	48,3093	48,3904	48,4716
120	48,9599	49,0416	49,1233	49,2051	49,2869
121	49,7794	49,8617	49,9439	50,0265	50,1089
122	50,6056	50,6885	50,7716	50,8547	50,9379
123	51,4386	51,5222	51,6059	51,6898	51,7737
124	52,2784	52,3627	52,4471	52,5316	52,6162
125	53,1249	53,2099	53,2951	53,3803	53,4655
126	53,9784	54,0641	54,1498	54,2357	54,3216
127	54,8386	54,9249	55,0114	55,0979	55,1845
128	55,7056	55,7926	55,8798	55,9669	56,0543
129	56,5794	56,6671	56,7546	56,8428	56,9308
130	57,4599	57,5484	57,6369	57,7255	57,8141
131	58,3474	58,4365	58,5256	58,6149	58,7042
132	59,2416	59,3313	59,4212	59,5111	59,6011
133	60,1426	60,2329	60,3236	60,4142	60,5049
134	61,0504	61,1415	61,2327	61,3239	61,4154
135	61,9649	62,0568	62,1487	62,2407	62,3327

APPENDIX.

247

Gallons from 91 to 136 Inches Diameter.

Diam. in ins.	.5	.6	.7	.8	.9
91	28,4656	28,5279	28,5902	28,6526	28,7149
92	29,0912	29,1541	29,2171	29,2802	29,3433
93	29,7236	29,7872	29,8509	29,9146	29,9785
94	30,3628	30,4271	30,4915	30,5559	30,6204
95	31,0088	31,0738	31,1388	31,2039	31,2691
96	31,6616	31,7273	31,7930	31,8588	31,9246
97	32,3212	32,3875	32,4539	32,5204	32,5869
98	32,9876	33,0546	33,1217	33,1888	33,2561
99	33,6608	33,7285	33,7963	33,8641	33,9320
100	34,3408	34,4092	34,4776	34,5461	34,6147
101	35,0276	35,0967	35,1658	35,2350	35,3042
102	35,7212	35,7909	35,8607	35,9306	36,0005
103	36,4216	36,4920	36,5625	36,6330	36,7037
104	37,1288	37,1999	37,2711	37,3423	37,4136
105	37,8428	37,9146	37,9864	38,0583	38,1303
106	38,5636	38,6361	38,7086	38,7812	38,8538
107	39,2912	39,3643	39,4375	39,5108	39,5841
108	40,0256	40,0994	40,1733	40,2472	40,3213
109	40,7668	40,8413	40,9159	40,9905	41,0652
110	41,5148	41,5899	41,6652	41,7405	41,8159
111	42,2696	42,3455	42,4214	42,4974	42,5734
112	43,0312	43,1077	43,1843	43,2609	43,3377
113	43,7996	43,8768	43,9541	44,0314	44,1089
114	44,5748	44,6527	44,7307	44,8087	44,8868
115	45,3568	45,4354	45,5139	45,5927	45,6715
116	46,1456	46,2249	46,3042	46,3836	46,4629
117	46,9412	47,0211	47,1011	47,1812	47,2613
118	47,7436	47,8242	47,9049	47,9856	48,0665
119	48,5528	48,6341	48,7155	48,7969	48,8784
120	49,3688	49,4503	49,5328	49,6149	49,6971
121	50,1916	50,2743	50,3570	50,4398	50,5226
122	51,0212	51,1045	51,1879	51,2714	51,3549
123	51,8576	51,9416	52,0257	52,1098	52,1941
124	52,7008	52,7855	52,8703	52,9551	53,0399
125	53,5508	53,6362	53,7216	53,8071	53,8927
126	54,4076	54,4937	54,5798	54,6659	54,7522
127	55,2712	55,3579	55,4447	55,5316	55,6185
128	56,1416	56,2289	56,3165	56,4039	56,4917
129	57,0188	57,1069	57,1951	57,2833	57,3716
130	57,9028	57,9916	58,0804	58,1691	58,2582
131	58,7936	58,8831	58,9726	59,0622	59,1518
132	59,6912	59,7813	59,8715	59,9618	60,0521
133	60,5956	60,6864	60,7773	60,8682	60,9593
134	61,5068	61,5983	61,6899	61,7815	61,8732
135	62,4248	62,5169	62,6092	62,7015	62,7939

APPENDIX.

A Table of the Areas of Circles in Wine

Diam. in. incs.	.0	.1	.2	.3	.4
136	62,8864	62,9789	63,0714	63,1641	63,2568
137	63,8146	63,9077	64,0009	64,0943	64,1877
138	64,7496	64,8434	64,9374	65,0314	65,1255
139	65,6914	65,7859	65,8805	65,9752	66,0699
140	66,6399	66,7352	66,8305	66,9250	67,0213
141	67,5954	67,6913	67,7872	67,8831	67,9794
142	68,5576	68,6541	68,7508	68,8475	68,9443
143	69,5266	69,6238	69,7212	69,8186	69,9161
144	70,5024	70,6003	70,6983	70,7964	70,8946
145	71,4849	71,5836	71,6823	71,7811	71,8799
146	72,4744	72,5737	72,6729	72,7725	72,8720
147	73,4706	73,5705	73,6706	73,7707	73,8709
148	74,4736	74,5742	74,6749	74,7758	74,8767
149	75,4834	75,5847	75,6861	75,7876	75,8892
150	76,4999	76,6019	76,7041	76,8063	76,9085
151	77,5234	77,6261	77,7288	77,8317	77,9346
152	78,5536	78,6569	78,7604	78,8639	78,9675
153	79,5906	79,6946	79,7988	79,9029	80,0073
154	80,6344	80,7391	80,8439	80,9488	81,0538
155	81,6849	81,7904	81,8959	82,0015	82,1071
156	82,7424	82,8485	82,9546	83,0609	83,1672
157	83,8066	83,9133	84,0202	84,1271	84,2341
158	84,8776	84,9849	85,0926	85,2002	85,3079
159	85,9554	86,0635	86,1717	86,2799	86,3884
160	87,0399	87,1488	87,2577	87,3667	87,4757
161	88,1314	88,2409	88,3504	88,4601	88,5698
162	89,2296	89,3397	89,4499	89,5603	89,6707
163	90,3346	90,4454	90,5564	90,6674	90,7785
164	91,4464	91,5579	91,6695	91,7812	91,8930
165	92,5649	92,6772	92,7895	92,9019	93,0143
166	93,6904	93,8033	93,9162	94,0293	94,1424
167	94,8226	94,9361	95,0498	95,1635	95,2773
168	95,9616	96,0758	96,1902	96,3046	96,4191
169	97,1074	97,2223	97,3373	97,4524	97,5676
170	98,2599	98,3756	98,4913	98,6071	98,7229
171	99,4194	99,5357	99,6519	99,7685	99,8849
172	100,5856	100,7025	100,8196	100,9367	101,0539
173	101,7586	101,8762	101,9939	102,1118	102,2297
174	102,9384	103,0567	103,1751	103,2936	103,4122
175	104,1249	104,2439	104,3631	104,4823	104,6015
176	105,3184	105,4381	105,5578	105,6777	105,7976
177	106,5186	106,6389	106,7594	106,8799	107,0005
178	107,7256	107,8466	107,9678	108,0890	108,2103
179	108,9394	109,0611	109,1829	109,3048	109,4268
180	110,1599	110,2824	110,4049	110,5275	110,6501

APPENDIX.

249

Gallons from 135 to 180 Inches Diameter.

Diam. in inches.	5.	.6	.7	.8	.9
136	63,3496	63,4425	63,5354	63,6284	63,7214
137	64,2812	64,3747	64,4683	64,5620	64,6557
138	65,2196	65,3138	65,4081	65,5024	65,5969
139	66,1648	66,2597	66,3547	66,4497	66,5448
140	67,1168	67,2124	67,3079	67,4037	67,4995
141	68,0756	68,1719	68,2682	68,3646	68,4610
142	69,0412	69,1381	69,2351	69,3322	69,4293
143	70,0136	70,1112	70,2089	70,3066	70,4045
144	70,9928	71,0911	71,1895	71,2879	71,3864
145	71,9788	72,0778	72,1768	72,2759	72,3751
146	72,9716	73,0713	73,1710	73,2708	73,3706
147	73,9712	74,0715	74,1719	74,2724	74,3729
148	74,9776	75,0786	75,1797	75,2808	75,3821
149	75,9908	76,0925	76,1943	76,2961	76,3979
150	77,0108	77,1132	77,2156	77,3181	77,4207
151	78,0376	78,1407	78,2438	78,3470	78,4502
152	79,0712	79,1749	79,2787	79,3826	79,4865
153	80,1116	80,2159	80,3205	80,4250	80,5297
154	81,1588	81,2639	81,3691	81,4743	81,5796
155	82,2128	82,3186	82,4244	82,5303	82,6363
156	83,2736	83,3801	83,4866	83,5932	83,6998
157	84,3412	84,4483	84,5555	84,6628	84,7701
158	85,4156	85,5234	85,6313	85,7392	85,8473
159	86,4968	86,6053	86,7139	86,8225	86,9312
160	87,5848	87,6939	87,8032	87,9125	88,0219
161	88,6796	88,7895	88,8994	89,0094	89,1194
162	89,7812	89,8917	90,0023	90,1129	90,2237
163	90,8896	91,0008	91,1121	91,2234	91,3349
164	92,0048	92,1167	92,2287	92,3407	92,4528
165	93,1268	93,2394	93,3520	93,4647	93,5775
166	94,2556	94,3689	94,4822	94,5956	94,7089
167	95,3912	95,5051	95,6191	95,7332	95,8473
168	96,5336	96,6482	96,7629	96,8776	96,9925
169	97,6828	97,7981	97,9135	98,0289	98,1444
170	98,8388	98,9548	99,0708	99,1869	99,3031
171	100,0016	100,1183	100,2349	100,3518	100,4686
172	101,1712	101,2885	101,4059	101,5234	101,6409
173	102,3476	102,4656	102,5837	102,7018	102,8201
174	103,5308	103,6495	103,7683	103,8871	104,0060
175	104,7208	104,8402	104,9596	105,0791	105,1987
176	105,9176	106,0377	106,1578	106,2780	106,3982
177	107,1212	107,2419	107,3627	107,4836	107,6045
178	108,3316	108,4530	108,5745	108,6960	108,8177
179	109,5488	109,6709	109,7931	109,9153	110,0376
180	110,7728	110,8956	111,0184	111,1413	111,2643

M 5

An explanation of the table of wine areas.

The explanation of the table of ale areas may serve to explain this also, because the diameters and areas are placed alike in both tables; for if you find any diameter from 1 to 180 inches in the first or seventh column, and the tenths (if any) at the head of the table, at the angle of meeting you will have the area required to the ten thousandth part of a wine gallon.

And in case the given diameter should exceed the limits of the table, then, according to the directions in the like case for ale areas, find the area of half the given diameter, which multiply by 4, and the product will be equal to the area required.

ART. V.

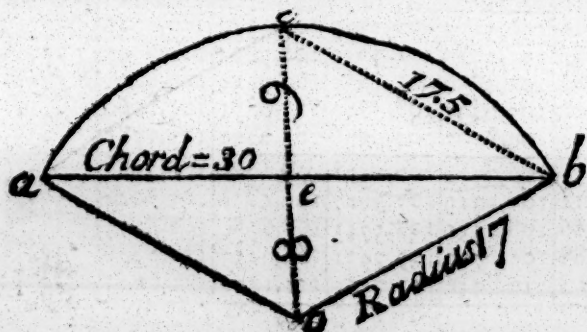
To find the area of a sector of a circle. See the following figure.

DEFINITION.

A Sector of a circle is a figure contained between the two semidiameters and part of the arch of a circle, and is equal in area to a triangle whose perpendicular is equal to the radius, and the base equal to the arch line of the sector.

RULE.

Multiply the radius or semidiameter by half the length of the arch, or the whole arch by the radius, and the product will be the area in inches; which divided by 282. 231. 2150.42, will give the area in ale and wine gallons, or malt bushels.



The length of the arch line not being easily obtained by measure, I shall give a practical method for finding the same, by having the length of the chord of half the arch given ; which will come pretty near the truth.

Of finding the area of the segment of a circle.

To find the length of the arch multiply the chord of half the arch by 8 ; from that product subtract the chord of the whole arch, and the remainder divide by 3, and the quotient will be nearly equal to the length of the arch.

EXAMPLE.

Let the radius $bo = ao = co = 17$ inches, the chord of the whole arch $ab = 30$ inches, and the chord of half the arch $bc = 17.5$ inches : to find the area of the sector $acbo$ in square inches.

OPERATION.

1. For the length of the arch acb .

$$\begin{array}{rcl} \text{Chord of } \frac{1}{2} \text{ the arch } cb & = & 17.5 \\ \text{Common multiplier} & = & 8 \end{array}$$

$$140.0$$

$$\text{Chd. of the whole arch sub. } 30$$

$$\text{Remainder divide by } 3 \quad 3)110.0(36.666 = \text{arch } cb.$$

2. For the area of the sector.

$$\text{Length of the arch } acb = 36.666$$

$$\text{Multiply by } \frac{1}{2} \text{ the radius } = 8.5$$

$$\begin{array}{r} 183330 \\ 293328 \\ \hline \end{array}$$

$$\text{Product} \quad \text{---} \quad = \quad 311.6610 = \text{area of sect. in sq. inches.}$$

ART. VI.

To find the area of the segment of a circle *See the last figure.*

DEFINITION.

THE segment of a circle is the part contained between the chord ab and the arch acb . Now it is plain from the figure, that if the area of the triangle abo be taken from the area of the sector found in the last article, the remainder will be equal to the area of the segment $acbe$.

RULE.

Find the area of the sector by the directions given in the last article; then find the area of the triangle abo , which subtract from the former, and the remainder will be the area of the segment required.

EXAMPLE.

Let the dimensions be the same as in the last article, viz. radius $ob = oc = 17$ inches, chord $ab = 30$ inches, chord $cb = 17.5$ inches, and the versed sine ec , or height of the segment $= 9$ inches, and the perpendicular height of the triangle $eo = 8$ inches: to find the area of the segment $acbe$ in inches.

OPERATION.

Base of the triangle $ab = 30$

$\frac{1}{2}$ Perpendicular $eo = 4$

—

Subtract the product $= 120 =$ area of triangle abo

From area of sector $= 311.661$

Remainder — $= 191.661 =$ area of segmt. sq. inches.

ART'

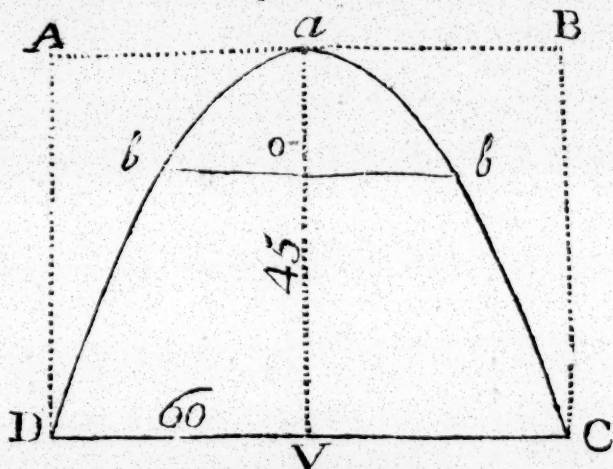
A R T. VII.

To find the area of a parabola. See the following figure.

A Parabola is one of the conic sections, and is formed by cutting a cone by a plane passing parallel to the opposite side of its entrance, and is equal to $\frac{2}{3}$ of the parallelogram that circumscribes it, viz. $D a c$ is equal to $\frac{3}{4}$ of $A B C D$.

R U L E.

Multiply the base or ordinate $D C$ by the perpendicular or axis $a v$, and that product divide by 1 and $\frac{1}{2}$ of the square divisor in page 57; the quotient will give the area of the same kind as your divisor.



E X A M P L E.

Let the ordinate or base $D C = 60$ inches, and the axis or perpendicular $a v = 45$ inches, what will the area be in ale gallons?

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Base } D C & \text{---} & = 60 \\
 \text{Perpendicular } a v & = & 45 \\
 & & \text{---} \\
 & & 300 \\
 & & 240 \\
 & & \text{---}
 \end{array}$$

$1\frac{1}{2}$ of 282 = 423)2700(6.38 = area in ale gallons.

A R T.

ART. VIII.

To find the content of a parabolic conoid.

DEFINITION.

A Parabolic conoid is generated by the rotation of the semiparabola $da v$ about its axe av , remaining fixed till the motion end where it first began; and is equal to half its circumscribing cylinder.

RULE.

Multiply the square of the diameter of the base by the height, and that product, divided by double of the circular divisors in page 57, will give the content in the same denomination as the divisor.

EXAMPLE.

Let the diameter of the base $dc = 60$ inches, (see the last figure) and the height $av = 45$ inches: to find the content in ale gallons.

OPERATION.

$$\text{Base } dc \text{ --- } = 60$$

60

$$\text{Square of } dc \text{ --- } = 3600$$

$$\text{Perpendicular } av \text{ --- } = 45$$

18000

14400

--- content.

$$2 \times 359.05 = 718.1) 162000 (225.59 \text{ ale gallons.}$$

ART. IX.

To find the content of the frustum of a parabolic conoid.

DEFINITION.

THE frustum of a parabolic conoid is the lower part of the conoid cut off at $b o b$ parallel to the base $d v c$. See the last figure.

RULE.

Multiply the sum of the squares of the diameters of the ends by the length, and that product, divided by double of the circular divisors in page 57, will give the content in the same kind as the divisor.

Or find the areas of the diameters and add them together, and multiply the sum by half the height; the product will be the content.

EXAMPLE.

Admit the frustum's height $= v o$ to be $= 30$ inches, and the least ordinate or diameter $b o b = 34.6$ inches, and the greatest $d v c = 60$ inches as before, what will its content be in ale gallons?

OPERATION.

$$\begin{array}{rcl}
 b o b = 34.6 & d v c = 60 & \\
 \underline{34.6} & \underline{60} & \\
 2076 & d v c \text{ sq.} = 3600 & \\
 1384 & d o d \text{ sq.} = 1197.16 & \\
 \underline{1038} & & \\
 1197.16 & \text{Sum} = 4797.16 & \\
 & v o = 30 &
 \end{array}$$

$$\begin{array}{rcl}
 718,1)143914,80(200,41 = \text{cont.} & & \\
 \underline{14362} & & \text{in ale gallons.}
 \end{array}$$

$$\begin{array}{r}
 \dots 29480 \\
 \underline{28724} \\
 \cdot 7560 \\
 \underline{7181} \\
 \cdot 379
 \end{array}$$

ART.

APPENDIX.

ART. X.

To find the content of a spheroid.

DEFINITION.

A Spheroid is generated by the rotation of a semi-ellipsis about its transverse diameter till the motion end where it began, and is equal to $\frac{2}{3}$ of its circumscribing cylinder.

RULE.

Multiply the square of the conjugate diameter by the transverse diameter, and that product divide by the globular divisors in Chap. viii. Art. vi. the quotient will be the content of the same kind as the divisor made use of.

EXAMPLE.

Let the ellipsis in Chap. vii. Art. x. represent the spheroid, whose transverse diameter is = 72 inches, conjugate diameter 50 inches: to find the content in ale gallons.

OPERATION.

Conjugate diam. D C	— =	50	
		50	
Square of conjugate diam.	=	2500	
Transverse diam. A B	— =	72	
		5000	
		17500	
			Content.

$$\frac{2}{3} \text{ of } 359.05 \text{ — } = 538.58)180000.(334.21 \text{ A. G.}$$

In this manner may the content of the four last figures be found in any other denominations, provided you make use of the proper divisors for that purpose.

APPENDIX.

ART. XI.

To find the solidity of the ungulæ or hoofs of the frustum of a cone, having the length of the greatest and least diameters, and also the depth given in inches.

Rule I. for the greater hoof.

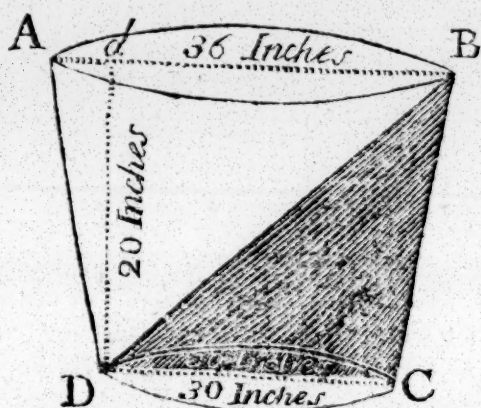
MULTIPLY the product of the greater diameter and the frustum's height by the square of the greater diameter made less by the product of the lesser diameter, into a mean proportional between the two diameters, and this last product divide by three times the circular divisors in page 57, multiplied into the difference of the diameters, and the quotient will be the solidity of the greater hoof.

Rule II. for the lesser hoof.

Multiply the product of the lesser diameter and height by the product of the greater diameter into a mean proportional between the two diameters made less by the square of the less diameter, and this last product divide by three times the circular divisors in page 57, multiplied into the difference of the diameters, and the quotient will be the solidity of the lesser hoof.

EXAMPLE.

Let the following figure represent the frustum of a cone, whose greater diameter $A B = 36$ inches, the lesser diameter $D C = 30$ inches, and the height $D d = 20$ inches; then the mean proportional will be nearly $= 32$ inches, and the difference of diameters $= 6$ inches: to find the solidity of the part $D A B$ the greater hoof, and also that of $D B C$ the lesser hoof in ale gallons.



OPERATION.

1. For the greater hoof D A B.

Gr. diam. A B =	36	Mean diam. =	32
Height D d =	20	Less dm. D C =	30
1 Prod. — =	720	2 Prod. — =	960
	216		
	108		
A B square =	1296		
2 Prod. sub. =	960		
Remains — =	336		
1 Prod. — =	720		
	6720		
	2352		
Last prod. =	241920		

PRODUCT.

$$3 \times 359 \times 6 = 6462)241920(37.43 = \text{content in ale gallons.}$$

$$\begin{array}{r} 19386 \\ \hline 48060 \\ 45234 \\ \hline 28260 \\ 25848 \\ \hline 24120 \\ 19386 \\ \hline \end{array}$$

Remains ——— . 4734

2. For the lesser hoof D B C.

$$\text{Less diam. } DC = 30 \text{ Gr. d. } AB = 36 \quad 30$$

$$\text{Height } Dd = 20 \text{ Mean dm.} = 32 \quad 30$$

$$1 \text{ Product} = 600 \quad 72 \text{ D c sq}^d = 900$$

$$\begin{array}{r} 108 \\ \hline \end{array}$$

$$2 \text{ Product} - = 1152$$

$$D c \text{ squared sub.} = 900$$

$$\text{Remains} - = 252$$

$$1 \text{ Product} - = 600$$

Content.

$$3 \times 359 \times 6 = 6462)151200(23.39 = \text{ale gallons.}$$

$$\begin{array}{r} 12924 \\ \hline 21960 \\ 19386 \\ \hline 25740 \\ 19386 \\ \hline 63540 \\ 58158 \\ \hline \end{array}$$

Remains ——— . 5382 Thus

Thus I have found the content of the greater hoof $DAB = 37.43$ ale gallons, and that of the lesser hoof $DBC = 23.39$ ale gallons, whose difference is 14.04 gallons; which exhibits the error of taking half the content of a cone's frustum for the solidity of the drip of a tun, which in fact is either the greater or lesser hoof of the frustum according to its position, whether it stands upon its greater or lesser base.

This method is very correct, and may be easily put in practice in finding the drip of a conical tun; for if you take a diameter at the base, and another parallel to that where the water cuts the side in the deepest place when the bottom is just covered, and also the perpendicular height, you will have a little frustum divided into two hoofs, when if the tun stands upon the lesser base the drip will be the lesser hoof, but if upon the greater base then the drip will be the greater hoof.

ART. XII.

To find the content of the ungulæ or hoofs of the frustum of a square pyramid, having the length of the greatest and least sides, and the depth given in inches.

Rule I. for the greater hoof.

1. **T**O twice the square of the greater side add the product of the greater and lesser sides, their sum multiply into the depth, and this product divide by 6 times the square divisors in page 57, the quotient will be the solidity of the greater hoof.

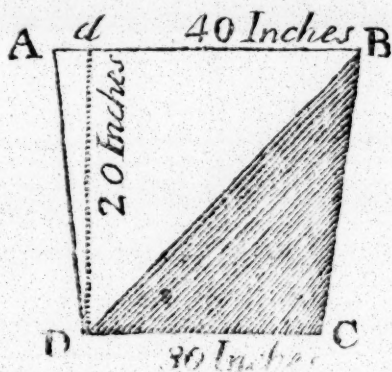
Rule II. for the lesser hoof.

2. **T**O twice the square of the lesser side add the product of the greatest and least sides, their sum multiply into the depth, and this product divide by 6 times the square divisors in page 57, the quotient will be the solidity of the lesser hoof.

EXAM-

EXAMPLE.

Let the following figure $A B C D$ represent the frustum of a square pyramid, whose greatest side $A B = 40$ inches, lesser side $D C = 30$ inches, and the depth $d D = 20$ inches: to find the content of the hoofs $D A B$ and $D B C$ in ale gallons.



1. For the greater hoof $D A B$.

OPERATION.

$$\begin{array}{rcl} A B & = & 40 \\ & & 40 \end{array} \quad \begin{array}{rcl} A B & = & 40 \\ D C & = & 30 \end{array}$$

$$\text{Square of } A B = 1600 \quad \text{Prod.} = 1200$$

$$\text{Twice sq. of } A B = 3200 \quad \text{Add} = 3200$$

$$\text{Sum} = 4400$$

$$\text{Multiply by } d D = 20$$

$$6 \times 282 = 1692 \quad \text{Content}$$

$$88000(52.01 = 8460 \quad D A B.$$

$$.3400$$

$$3384$$

$$.. 1600$$

$$1692$$

$$92 \text{ Too much.}$$

$$2. \text{ For}$$

2. For the lesser hoof D B c.

OPERATION.

D B	—	=	30		A B	=	40
			30		D C	=	30
			—				—
Square of D C	—	=	900		Prod.	=	1200
			2		Add	=	1800
			—				—
Twice sq. of D C	=	1800				3000	
			Multiply by d D	=		20	
						—	Content.
			6 × 282 = 1692			60000	(35.46 =
						5076	D B c.
						—	
						.9240	
						8460	
						—	
						.7800	
						6768	
						—	
						10320	
						10152	
						—	
Remains	—					.. 168	

The content of the greater hoof D A B = 52.01 added to the content of the lesser hoof D B c = 35.46 is equal to 87.47 ale gallons, equal to the whole content of the frustum A B C D.

These two rules will be of good use in finding the drip of any square pyramidal tun; for if the tun stands flanting upon the greater base, then the drip will form the greater hoof; but if upon the lesser base, the drip will form the lesser hoof of a frustum, cut off where the water touches the side, in the deepest place when the bottom is just covered. Therefore, if the dimensions of the sides be taken between every 6 or 10 inches of the depth, agreeable to the directions given in Chap. x.

Art.

Art. vi. for a conical tun, and the drip be found according to the rules given in this article, then any tun of this form may be easily inched, and tabulated, so as to give the content at any depth.

ART. XIII.

To gage any irregular, elliptical, or oval vessel, by taking a competent number of equidistant ordinates.

I Have already laid down rules, and given sufficient examples, for gaging all sorts of regular figures, whether they be superficies or solids; but since in practice we frequently meet with ovals of an irregular form, such as brewers tuns, distillers backs, &c. which contain more than the ellipsis, and less than a circle, consequently their true areas cannot be found by any of the preceding rules; therefore that nothing may be wanting, it will be necessary here to shew how this may be effected, whenever such cases shall occur.

In order to do this I shall have recourse to the method of equidistant ordinates, deduced from Simpson's Dissertation, page 109; which finds the area of any space comprehended between two perpendicular right lines, and the curve that joins them; observing, at the same time, that former editions of this work, laid down a different method; but the honourable Excise Board, by a general letter of July 1775, having adopted a general rule from the above ingenious author's approximation, for finding the area of curvilinear vessels; by adding the area of the segments, included between the extreme ordinates and the vertex, to the area found within the said ordinates, making together the whole area of the bounding curve; it becomes necessary to explain the use and manner of the same, that this book may accord with the practice of the Excise, for whose use it was principally intended.

The area of an irregular oval cannot be obtained otherwise than by ordinates; for let the curvature be what

what it will, whether greater or less than an ellipsis, nay, if it should be found equal to an ellipsis, or even a true circle, yet it will hold good, and the area will be found properly true: therefore, as there may be curves that greatly exceed the elliptical, and the general rules to be given holds good in all, no areas should be calculated by the transverse and conjugate diameters only; as either the trader or revenue may greatly suffer, if it should not happen to be a true ellipsis; of which, no doubt, the Excise Board was fully aware, by ordering curvilinear vessels to be thus gaged; as well to avoid the tediousness of rules hitherto laid down, which though demonstrably accurate, yet the simplicity of this approximation must be more eligible to practical gagers, as being sufficiently true for the purposes here applied.

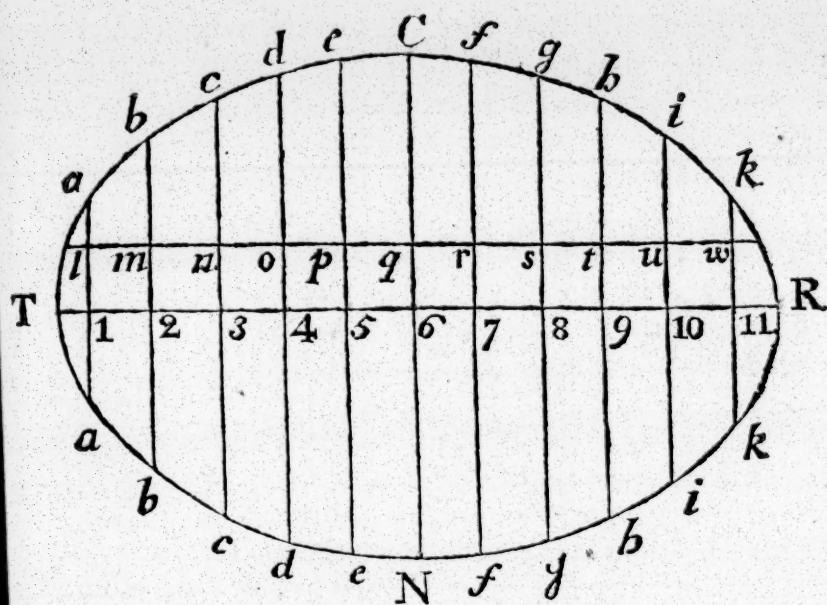
The more irregular the figure, the more ordinates should be taken; for the more we take, the nearer we shall approach to truth, without much increase of trouble in computing the exact gage; which has been the great complaint of all former methods when the ordinates were numerous, as the rules laid down not being general, but particular rules for the number of ordinates made use of.

EXAMPLE I.

Let the following figure represent the bottom of the tun, or wash back to be gaged, not elliptical but of an unknown-form.

With a chalk line, held by an assistant, strike the longest or transverse diameter TR , which will be easily found by your gaging-rods; and at right angles, perpendicular thereto, exactly in the middle G , strike the greatest ordinate or conjugate CGN ; which quartering the tun as sufficiently accurate for the purpose, as by any other mode laid down in other writers of gaging, for finding the transverse and conjugate.

The



Then, with a pair of coopers (or large) compasses, set off from their intersection or center 6, both ways, two, three, four, or five, &c. equal distances; according as you intend to take five, seven, nine, or eleven, &c. ordinates, which number of ordinates, are to be always *odd*; therefore if you divide TR with some even number, one less than the number of intended ordinates, the quotient will be their equidistance; and the remainder the measure of the segments TI and IR both together; which should be so contrived that TI and IR (both equal) be as little as possible. Thus TR being 109.2 inches, that, divided by ten, as eleven ordinates are intended, gives, in the quotient, ten inches for the ordinates distance from each other: and the remainder 9.2 inches is the sum of the segments TI , and IR , viz. 4.6 inches each; then, with ten inches, open your compass, beginning from the center 6, both ways, to 5, 4, 3, 2, 1, 7, 8, 9, 10, 11, make marks in TR ; and with the same extent, set off from 6, upon lw drawn parallel to TR , the same number of distances, all equal to each other; with the chalk line, through the points $l1$, $m2$, $n3$, &c. to $w11$, strike the

N

the

the ordinates $a a$, $b b$, $c c$, &c. to $k k$; which eleven ordinates are asunder, or equidistant, ten inches, and perpendicular to $T R$, leaving the two segments $T a a$, and $R k k$ in height, or $T I$ and $I I R$ each equal to 4.6 inches.—Another parallel line to the transverse, like $l w$, may be drawn on the other side, if the vessel is large, in order to be more certain the ordinates are perpendicular, as there will be three points to strike the ordinates through, instead of two. Now as the tun or wash back is to be inched, and tabled like other tuns (see p. 136) at 6, 8, 10, 12, &c. inches of the depth, by taking ordinates between, take a chalked plumb line, and hold at the top of the tun or back so as to directly fall or hang freely over the ordinate $a a$; then your assistant holding the plummet at the extremity a , strike a line against the side from thence to the top, where the line touched or marked when the plummet was directly over the ordinate $a a$: proceed in the same manner with the other ordinates, $b b$, $c c$, $d d$, $c n$ to $k k$, and R , striking lines up the side, from their extremities b , c , d , &c. round to T : then is your tun, or back, ready for taking dimensions, except marking on each line struck up the side, the distances of 6, 8, 10, 12, &c. inches of the depth; between which ordinates are to be taken in order to find the whole content for tabling: when the lengths of the several ordinates 1, 2, 3, 4, 5, 6, &c. to 11, at the several depths, are taken, they are rendered more ready for computation if put down in the following form; remembering that after any transverse is found, the difference between its length and sum of ordinates (here 100) makes the numbers in the column called sum of segments; or $T I = I I R$ in one sum.

Mr. ——— 2 Tun. (or Back) raised Nov 8 1780

Mr. ——— 2 Tun, (or Back) gaged Nov. 8, 1779.

11 Ordinates, 10 Inches equidistant = 100 Inches.

[illegible]

N. B. When a diffiller's wash back is gaged, the contents must be kept in gallons.—Likewise the distance between the extreme ordinates 1 and 11, in contriving the number of ordinates, be somewhat less than the whole transverse at top of the tun or back, if they stand on the greater end, in order to keep the segments within the top curve.

1st. General rule for finding the area of any curvilinear plane, bounded at each end with an ordinate, by dividing the same into any odd numbers of equidistant and parallel ones.

To the sum of the first and last, or the extreme ordinates, add four times the sum of the 2nd, 4th, 6th, 8th, &c. or *even* ordinates; and also double the sum of the rest, not included in the above; which will be the 3rd, 5th, 7th, 9th, &c. or *odd* ordinates; this total, multiplied by $\frac{1}{3}$ of the ordinates equidistance, or by the equidistance, and the last product, divided by $\frac{1}{3}$, the quotient in this, or product in the former, will be the required area between the extreme ordinates.--This, divided by 282, 231, 2150, will be the area in ale and wine gallons and malt respectively.

Before we proceed to computation of the tun in question, a rule must be laid down, to find the area of the two segments $T a a$ and $k k R$; which segments being so small in respect to the whole figure, it is immaterial whether they be considered as parabolas, or segments of circles: but since parabolas are $\frac{2}{3}$ of the circumscribing parallelogram (see p. 253,) it will render a very easy rule to find their area, in respect to that of segments of circles.—Therefore, instead of finding the area of the two segments $T a a$ and $k k R$, separate, the following rule finds their area together very readily.

2. General rule for finding the area of the segments, included between each of the extreme ordinates, and adjoining curve.

Multiply the sum of the extreme ordinates by the sum of their distance from the curve $\frac{1}{3}$ of the product will be the area of both; this, divided by 282, 231, 2150, will be the area in ale and wine gallons and malt respectively: but as the areas found by this and the former, or 1st general rule, added together, is to be the *whole* area of the vessel, there will be no occasion to make use of the divisors until both are added together as follows:

OPERA-

OPERATION.

By the first general rule we have the sum	Inches.
of the two extreme ordinates (here	
N ^o 1 and 11) — — — — — }	49.3
Four times the sum of N ^o 2nd, 4th, 6th,	
8th, 10th, ordinates, — — — — — }	1266.4
Twice the sum of all the rest, viz. 3rd, }	
5th, 7th, 9th, — — — — — }	552.2
	1867.9

Which being multiplied by 10, the ordinates }
distance, and divided by 3 (as $\frac{1}{3}$ of 10 is } 626.233
not to be exactly had) becomes — }

Which is the area of the space, comprehended by the extreme ordinates and bounding curve, viz. *a, b, c, d, e, c, f, g, h, i, k, k, i, &c.* round to *a*, which, when the segments *T a a* and *k k R* is added, will be the whole area of the curvilinear vessel at that depth from bottom, or any depth thus computed.

Now by rule 2nd, the sum of the extreme ordinates being 49.3 inches, and by the table the sum of segments being 4.8 inches; these two sums, multiplied together, and divided by 3, give — — — — — }

78.88

Whole area 6305.21

This, divided by 282, gives 22.36 gallons, as in the Table, p. 267; and this area, multiplied by the depth 10 inches where taken between, and divided by 34, gives 6b. 2f. 2.6g.; and so all the rest of the areas was worked to make out the whole content.—By which it appears how extremely easy any curvilinear vessel may be gaged, let it be ever so irregular in the

curve, and with what little trouble, even if the ordinates are numerous: having, therefore, computed each depth, a table of the utensil is to be made in the same manner as the example in p. 136.—Farther examples are no way necessary; if the inquisitive gager should wish to know the demonstration of this rule, and how near this approximation comes to exact truth, see Simpson's Dissertations, and Hutton's Mensuration, where fundry examples are given with ordinates drawn in a circle, ellipsis, and hyperbola, to shew its great nearness to other methods and utility.

ART. XIV.

To find the contents of the three first varieties of casks standing on their heads, at every inch of their depth, by having their head and bung diameters, and length in inches, given, which is commonly called the inching of casks.

1st. For the first variety.

RULE.

1st. **D**IVIDE $\frac{1}{3}$ of the difference of the areas of the head and bung diameters by the square of half the length, and the quotient will be a common multiplier.

2. From the area of the bung diameter subtract the product of the square of the distance from the bung into the multiplier, and the remainder multiply by the distance from the bung, and the product will be the content required.

2d. For

2d. For the second variety.

RULE.

Divide $\frac{7}{8}$ of the difference of the areas of the head and bung diameters by the square of half the length of the cask, and the quotient will be a common multiplier for the second variety, which must be managed in the same manner as that given for the first variety.

3d. For the third variety.

Divide $\frac{4}{9}$ of the difference of the areas of the head and bung diameters by the square of half the length of the cask, and the quotient will be a common multiplier for the third variety; which managed according to the rule given for the first variety, will give the content at any inch of its depth.

But this will appear more plain, by a few examples which I shall exhibit, in inching of a cask of the same dimensions, with respect to head, bung, and length, supposing it to belong to either of the three first varieties.

1st. Example for the first variety.

Admit the length of the cask to be 28 inches, bung diameter 27.2 inches, and head 22.6 inches, to find how many wine gallons it will contain on every inch of its depth, when it stands upon one of its heads.

APPENDIX.

OPERATION.

$\frac{1}{2}$ Len.	Inch.	Area.	Factor.
14	B. diameter 27.2 =	2.5155	
—	H. diameter 22.6 =	1.7366	
56		<u> </u>	
14		$\frac{1}{2}$)7789	
<u> </u>		<u> </u>	
Sq. = 196		196).259633(.0013246	
		<u>196</u>	
		636	
		<u>588</u>	
		483	
		<u>392</u>	
		913	
		<u>784</u>	
		1293	
		<u>1176</u>	
		117	

Then for the first inch from the bung.

From the area of the bung diameter	=	2.5155
Subtract .0013246 $\times$ 1 squared	=	<u>.0013246</u>
Content of the first inch	— =	2.5141754

For the second inch.

From the area of the bung diameter	=	2.5155
Subtract .0013246 $\times$ 2 squared	=	<u>.005298</u>
Remainder	— =	2.510202
		<u>2</u>
Content at the second inch	— =	5.020404
		for

For the third inch.

From the area of the bung diameter	=	2.5155
Subtract .0013246 × 3 squared —	=	.011921
Remainder —	=	2.503579
		3
Content at the third inch —	=	7.510727

For the fourth inch.

From the area of the bung diameter	=	2.5155
Subtract .0013246 × 4 squared —	=	.021193
Remainder —	=	2.494307
		4
Content at the fourth inch —	=	9.977228

Having found the content of the four first inches, the rest are easily found by adding and subtracting the differences; for the third differences being all equal throughout the frustum, by finding the first, it will suffice for all the rest. As for example, the third difference in this case is .0079, which added to .0238, will give .0317 for the third of the second difference; which subtracted from 2.4665, the third first difference, will give 2.4368 for the fourth first difference; which added to the content of the fourth inch, before found, will give the content of the five first inches from the bung; as will appear from the ensuing table. And in this manner proceed, till you come to half the length of the cask.

A Table for the first Variety.

Inches from bung.	Contents.	1st Diff.	2d Diff.	3d Diff.
1	2.5142	2.5062	.0159	.0079
2	5.0204	2.4903	.0238	.0079
3	7.5107	2.4655	.0317	.0079
4	9.9772	2.4348	.0396	.0079
5	12.4120	2.3952	.0475	.0079
6	14.8072	2.3477	.0554	.0079
7	17.1549	2.2923	.0633	.0079
8	19.4472	2.2290	.0712	.0079
9	21.6762	2.1578	.0791	.0079
10	23.8340	2.0787	.0870	.0079
11	25.9127	1.9917	.0949	.0079
12	27.9044	1.8968	.1028	
13	29.8012	1.7940		
14	31.5952			

The above contents are the solidities at every inch from the bung to the head ; which being found, the contents on every inch of the lower frustum may be had, by continually adding of the first differences, beginning at the farthest inch from the bung, and so proceed upwards. And the content at every inch of the upper frustum is obtained, by adding of the first differences to half the content of the cask, beginning at the first inch from the bung, and going downwards. An instance of which I shall at large exhibit in the table for the second variety.

2d. Example for the second variety.

Supposing the cask to be inched were of the same dimensions as that in the last example, and to belong to the second variety ; to find its content on every inch in wine gallons.

OPERA-

APPENDIX

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OPERATION.

Inch. Areas.
 Bung diameter = 27.2 = 2.5155
 Head diameter = 22.6 = 1.7366

.7789
 7

18) 5.4523(.3029
 54

$\frac{1}{2}$ Length = 14
14
 —
 56
14

52
 36
 —
 163
 162
 —
 1

196).3029(.001545 = multiplier.

196
 —
 1069
 980
 —
 890
 784
 —
 1060
 980
 —
 80

For the first inch.

From the area of the bung diameter = 2.5155
 Subtract .001545 $\times$ 1 squared — = .001545
 Content of the first inch — = 2.513955
 N 6 For

For the second inch.

From the area of the bung diameter	=	2.5155
Subtract .001545 × 2 squared —	=	0.00618
		2.50932
		2

Content of the second inch —	=	5.01864
------------------------------	---	---------

For the third inch.

From the area of the bung diameter	=	2.5155
Subtract .001545 × 3 squared —	=	0.013905
		2.501595
		3

Content of the third inch —	=	7.504785
-----------------------------	---	----------

For the fourth inch.

From the area of the bung diameter	=	2.5155
Subtract .001545 × 4 squared —	=	0.02472
		2.49078
		4

Content of the fourth inch —	=	9.96312
------------------------------	---	---------

Having found the contents of the four first inches, the rest will be had by adding and subtracting of the differences, as in the last example; which, when done, will be as in the following table.

A Table for the second Variety.

Inches from bung.	Contents.	1st Diff.	2d Diff.	3d Diff.
1	2.5139	2.5047	.0185	.0094
2	5.0186	2.4862	.0279	.0094
3	7.5048	2.4583	.0373	.0094
4	9.9631	2.4210	.0467	.0094
5	12.3841	2.3743	.0561	.0094
6	14.7584	2.3182	.0655	.0094
7	17.0766	2.2527	.0749	.0094
8	19.3293	2.1778	.0843	.0094
9	21.5071	2.0935	.0937	.0094
10	23.6006	1.9998	.1031	.0094
11	25.6004	1.8967	.1125	.0094
12	27.4971	1.7842	.1219	
13	29.2813	1.6623		
14	31.			

As the contents found in the two preceeding tables begin from the bung to the head diameters, they will not answer the end, so as to shew the content on every inch of a standing cask, without a farther operation. Therefore I shall, in this example, according to my promise, shew the contents on every inch, in gallons and decimal parts, from the bottom to the top of a standing cask; and also collect the same to the nearest even gallon; which latter will be the only necessary numbers from the whole process to insert in the table-book.

The first differences being continually added for the lower frustum, beginning from the bottom and proceeding upwards, will shew the content at every inch of the lower half of the cask; and by adding of first differences to the half content of the cask, beginning at the top and proceeding downwards, you will have the content on every inch of the upper half of the cask; which being done, it will stand as in the ensuing table.

A Table shewing the Content of a Cask of the second Variety on every Inch of its Depth.

Inch.	Contents.	Contts. in even galls.	Inch.	Contents.	Contts. in even galls.
1	1.6623	2	15	33.5139	34
2	3.4465	3	16	35.0186	36
3	5.3432	5	17	38.5048	39
4	7.3430	7	18	40.9631	41
5	9.4365	9	19	43.3841	43
6	11.6143	12	20	45.7584	46
7	13.8670	14	21	48.0766	48
8	16.1852	16	22	50.3293	50
9	18.5595	19	23	52.5071	53
10	20.9805	21	24	54.6006	55
11	23.4388	23	25	56.6004	57
12	25.9250	26	26	58.4971	58
13	28.4297	28	27	60.2813	60
14	31.	31	28	62.	62

3d. Example for the third variety.

Supposing the cask to be inched were of the third variety, and the dimensions the same as before, viz. head 22.6, bung 27.2, and length 28 inches; to inch and tabulate the same in wine gallons.

APPENDIX.

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OPERATION.

	Inches.	Area.
From area of B. diameter 27.2	=	2.5155
Substr. area of H. diameter 22.6	=	1.7366
$\frac{1}{2}$ Length = 14		
14		.7789
		4
56		
14		9)3.1156 multiplier
		196) .346177(.001766
196		196
		1501
		1372
		1297
		1176
		1217
		1176
		41

For the first inch.

From the area of the bung diameter	=	2.5155
Subtract .001766 $\times$ 1 squared	— =	0.001766
Content of the first inch	— =	2.513734

For the second inch.

From the area of the bung diameter	=	2.5155.
Subtract .001766 $\times$ 2 squared	— =	0.007064
		2.508436
		2
Content of the 2d inch	— =	5.016872

For

For the third inch.

$$\begin{array}{rcl}
 \text{From the area of the bung diameter} & = & 2.5155 \\
 \text{Subtract } .001766 \times 3 \text{ squared} & - & = 0.015894 \\
 & & \hline
 & & 2.499606 \\
 & & 3
 \end{array}$$

$$\text{Content of the third inch} \quad \text{---} \quad = 7,498818$$

For the fourth inch.

$$\begin{array}{rcl}
 \text{From the area of the bung diameter} & = & 2.5155 \\
 \text{Subtract } .001766 \times 4 \text{ squared} & - & = 0.028256 \\
 & & \hline
 & & 2.487244 \\
 & & 4
 \end{array}$$

$$\text{Content of the fourth inch} \quad \text{---} \quad = 9,948976$$

The contents of the four first inches being found, the rest will be found as before, by adding and subtracting the differences; which being so easy, it will be quite needless to add any thing further on this head, therefore I shall conclude with the following table, from whence the cask may be inched and tabulated, just in the same manner as was shewn for the second variety.

To
le
m

M
that
off t
cont
and
whic
multi
If
have
2150,

A Table for the third Variety..

Inches from bung.	Contents.	1st Diff.	2d Diff.	3d Diff.
1	2.5137	2.5031	.0211	.0107
2	5.0168	2.4820	.0318	.0107
3	7.4988	2.4502	.0425	.0107
4	9.9490	2.4077	.0532	.0107
5	12.3567	2.3545	.0639	.0107
6	14.7112	2.2906	.0746	.0107
7	17.0018	2.2160	.0853	.0107
8	19.2178	2.1307	.0960	.0107
9	21.3485	2.0347	.1067	.0107
10	23.3832	1.9280	.1174	.0107
11	25.3112	1.8106	.1281	.0107
12	27.1218	1.6825	.1388	
13	28.8043	1.5437		
14	30.3480			

ART. XV.

To find the content of a couch or floor of malt, having the length, breadth, and depth given in inches, by a new method.

RULE.

MULTIPLY half the length of the floor by the breadth, or half the breadth by the length; and that product by the depth; from this last product cut off three figures to the right hand, and it will give the content of the floor seven bushels too much in every 100, and so in proportion for a greater or lesser quantity; which excess may be deducted either by subtraction or multiplication.

If the malt divisor had been 2000, this method would have answered without any deduction; but since it is 2150, the multiplying by half the length or breadth will

will give too much, it being in effect only dividing by 2000 : to remedy this, if you multiply the product of the length, breadth, and depth, by .93, you will find the true content.

For as $2150 : 2000 :: 1 : .93 = \text{multiplier.}$

Or if you subtract seven bushels for every 100, and .7 for every 10 bushels, or .07 for every single bushel, you will have the content as before.

EXAMPLE.

Admit the length of the floor 400 inches, breadth 215, and depth 4 inches ; to find the content.

$$\begin{array}{rcl} \text{Breadth} & - & = 215 \\ \text{Half the length} & = & 200 \end{array}$$

$$\begin{array}{rcl} \text{Depth} & - & = 4 \end{array}$$

$$\begin{array}{rcl} \text{Product} & - & = 172. \\ \text{Product} & = & 172. \end{array}$$

$$\begin{array}{r} .93 \\ \hline 516 \\ 1548 \\ \hline \end{array}$$

159.96 = content in bushels.

By subtraction.

$$\begin{array}{rcl} \text{Product} & = & 172. \\ \text{Subtract} & = & 12.04 \end{array}$$

159.96 = content as above.

For 100 deduct 7. bushels.

For 70 deduct 4.9

For 2 deduct .14

$$\begin{array}{rcl} \text{Sum} & - & = 12.04 \end{array}$$

This

This method of casting of malt gages is so very easy, that it appears quite needless to add any more examples.

And I think it will be of use to officers in making compares, before they enter the gages in their books, and may be a means of preventing many mistakes.

ART. XVI.

To money cistern or couch bushels in a short and ready manner.

RULE.

MULTIPLY the two first figures to the right hand by 4, and the product will be shillings and tenths; the rest multiply by 2, the product will be pounds.

EXAMPLE I.

What is the duty of 4562 couch bushels?

OPERATION.

$$\begin{array}{r}
 45\overline{)62} \\
 \underline{2} \quad 4 \\
 91.4.8 \\
 \underline{12} \\
 9.6 \\
 \underline{4} \\
 2.4
 \end{array}$$

Answer 91 l. 4 s. 9 d. $\frac{1}{2}$.4.

APPENDIX.

EXAMPLE II.

To find the duty of 3268 couch bushels.

$$\begin{array}{r} 32 \overline{) 68} \\ 2 \overline{) 4} \end{array}$$

$$\begin{array}{r} 65.7.2 \\ 12 \end{array}$$

$$\begin{array}{r} 2.4 \\ 4 \end{array}$$

$$\begin{array}{r} 1.6 \end{array}$$

Answer — 65l. 7s. 2d. $\frac{1}{4}$.6. = old duty.

Half of which = 32l. 13s. 7d. $\frac{1}{4}$ = new duty.

And at the request of several friends in the excise, and for the sake of pupils and young officers, I have at the latter end given precedents of accounts made up, viz. for excise, malt, candles, soap, hides, and paper, and have drawn an abstract from the whole; which are so made up and titled against every fitting on large sheets of paper ruled for that purpose, according to the following specimens.

E X C I S E.

_____ Collection.

_____ Division.

Fourth Round Voucher.

From 11ber 10th, to 12ber 22nd, 1768.

x = vi = Cyder.			l.	s.	d.
20	= 78 $\frac{3}{4}$	= Common-Brewers	--	47	9 3
26 $\frac{1}{2}$	= 88 $\frac{1}{2}$	= 17 $\frac{3}{4}$ Victuallers	-- --	62	8 4
Total			--	109	17 7

CASH, one hundred and nine pounds seventeen shillings
and seven pence.

Collection. Division. Fourth Round

		C. Brewers.			
		x.	VI.		
November	9	30	$18\frac{1}{2}$	November	10
	13	$28\frac{1}{2}$	$17\frac{1}{2}$		17
	17	$31\frac{1}{2}$	19	Odds	
	24	$29\frac{1}{2}$	23	Daniel Wright	— —
Odds		$\frac{1}{2}$	$\frac{3}{4}$		
James Humphreys	120		$78\frac{3}{4}$	November	14
					20
				Odds	
				George Gale	— —
				November	15
					20
				Odds	
				Anne Leach	— —
				November	24
					30
				Odds	
				Thomas Cart	— —
				December	10
					17
				Odds	
				John Jones	— —
				December	12
				Odds	
				William Stokes	— —
				By Ales	— —
		120	$78\frac{3}{4}$		

Excise Voucher from qber 10th, to 10ber 22d, 1768.

Vicuallers.									
x.	vi.	Cyd.		x.	vi.	Cyd.			
9 $\frac{1}{2}$	7 $\frac{1}{2}$	1 $\frac{1}{2}$							
10 $\frac{1}{2}$	8 $\frac{1}{2}$								
20 $\frac{1}{2}$	16	1 $\frac{1}{2}$							
8 $\frac{3}{4}$	5	2							
9 $\frac{1}{4}$	5 $\frac{1}{2}$	2 $\frac{1}{2}$							
18 $\frac{1}{4}$	10 $\frac{1}{2}$	4 $\frac{1}{2}$							
14 $\frac{1}{2}$	9 $\frac{1}{4}$	1 $\frac{1}{2}$							
15 $\frac{1}{2}$	8 $\frac{3}{4}$	1 $\frac{1}{2}$							
30 $\frac{1}{2}$	18	3	1st Col.	120	78 $\frac{3}{4}$		l.	s.	d.
14 $\frac{1}{4}$	10	1 $\frac{1}{2}$	2d Col.	126 $\frac{1}{2}$	88 $\frac{1}{2}$	17 $\frac{3}{4}$	47	9	3
13 $\frac{1}{4}$	10 $\frac{1}{2}$	2 $\frac{1}{2}$	Total	—	—	—	62	8	4
28 $\frac{1}{4}$	20 $\frac{1}{2}$	4					100	17	7
7 $\frac{3}{4}$	5	1	The amount of this Voucher is one hundred and twenty barrels of Common Brewers strong, and seventy-eight barrels and three firkins of small beer. One hundred twenty-six barrels and a half of Vicualler's strong, eighty-eight barrels and half of small beer, and seventeen hogheads and three fourths of cyder. Cash, one hundred nine pounds seven-teen shillings and seven pence.						
6 $\frac{1}{2}$	5	1 $\frac{1}{2}$							
14 $\frac{1}{2}$	10	2 $\frac{1}{2}$							
10 $\frac{3}{4}$	9	2 $\frac{1}{4}$							
10 $\frac{3}{4}$	9	2 $\frac{1}{4}$							
3 $\frac{3}{4}$	4 $\frac{1}{2}$								
126 $\frac{1}{2}$	88 $\frac{1}{2}$	17 $\frac{3}{4}$							

Wm. Symons

MALT

C Y D E R 1766.

————— Collection.

————— Division.

Fourth Round Voucher.

From 9ber 10th, to 10ber 22nd, 1768.

Hhds.

17 $\frac{3}{4}$

L. s. d.

5 6 6

CASH, five pounds fix shillings and fix pence.

<i>Collection.</i>		<i>Division.</i>	<i>Fourth Round</i>
		No. of hhds.	
November 10		$1\frac{1}{2}$	
Odds			
Daniel Wright —		$1\frac{1}{2}$	
November 14		2	
20		2	
Odds		$\frac{1}{2}$	
George Gale —		$4\frac{1}{2}$	
November 15		$1\frac{1}{2}$	
20		$1\frac{1}{2}$	
Odds			
Ann Leach —		3	
November 24		$1\frac{1}{2}$	
30		2	
Odds		$\frac{1}{2}$	
Thomas Cart —		4	
December 10		1	
17		1	
Odds		$\frac{1}{2}$	
John Jones —		$2\frac{1}{2}$	
December 12		2	
Odds		$\frac{1}{4}$	
William Stokes		$2\frac{1}{4}$	
		$17\frac{3}{4}$	

APPENDIX.

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Cyder 1766 Voucher from 9ber 9th, to 10ber 22d, 1768.

	No. of hhds.	L.	s.	d.
Collection	17 $\frac{3}{4}$	5	6	6

The amount of this Voucher is seventeen hogsheds and three-fourths of cyder.

Cash is five pounds six shillings and six pence.



Wm. Symons.

B

93

C
A

M A L T.

— Collection.

— Division.

Fourth Round Voucher.

From 9ber 10th, to 10ber 22d, 1768.

Best.	Floor.	Cyder.	O	Addition.
			L. s. d.	L. s. d.
930	324	18 $\frac{3}{4}$	26 8 9	11 6 6

CASH, old duty twenty-six pounds eight shillings.
Additional duty eleven pounds six shillings and six pence.

Collection. Division. Fourth Round

	Bushels.		Hhds. Cyder.
	Best.	Floor.	
November 10	100	—	—
17	101	—	—
20	—	161	—
25	102	—	—
Odds	5	—	—
Thomas Wheeler—	308	161	—
November 12	105	—	—
15	103	—	—
17	—	162	—
20	102	—	—
Odds	2	1	—
John Denbigh —	312	163	—
November 16	104	—	—
20	103	—	—
27	101	—	—
Odds	2	—	—
James Humphrey —	310	—	—
George Hale Dealer	—	—	1
Daniel Wright —	—	—	1½
George Gale —	—	—	4½
Ann Leach —	—	—	3
Thomas Cart —	—	—	4
John Jones —	—	—	2½
Williams Stokes —	—	—	2¼
	930	324	18½

Malt Voucher from 9ber 10th, to 10ber 22d, 1768.

	Best.	Floor.	Cyder.	Old duty	Additi- onal.	
	5	8				
Column	920	324	18 $\frac{3}{4}$	26 8 0	11 6 6	

The amount of this Voucher is nine hundred and thirty bushels from Best, three hundred twenty-four from Floor, and eighteen hogheads and three fourths of Cyder.

Cash, old duty is twenty-six pounds eight shillings, new duty eleven pounds six shillings and six pence.

Wm. Symons.

W
4

C

C A N D L E S.

———— Collection.

———— Division.

Fourth Round Voucher.

From 9ber 10th, to 10ber 22nd, 1768.

Weight		L.	s.	d.
4894 at one penny per pound	—	20	7	10

CASH, twenty pounds seven shillings and ten pence.

Collection. Division. Fourth Round

		No. of pounds.
November	8	240
	10	255
	12	370
	13	195
	14	640
	16	120
Charles Wheeler		1820
November	11	195
	12	236
	14	455
	16	372
	19	139
	20	245
	23	362
	24	185
	25	672
	27	12
	28	12
	30	189
James King		3074
		4894

APPENDIX.

299

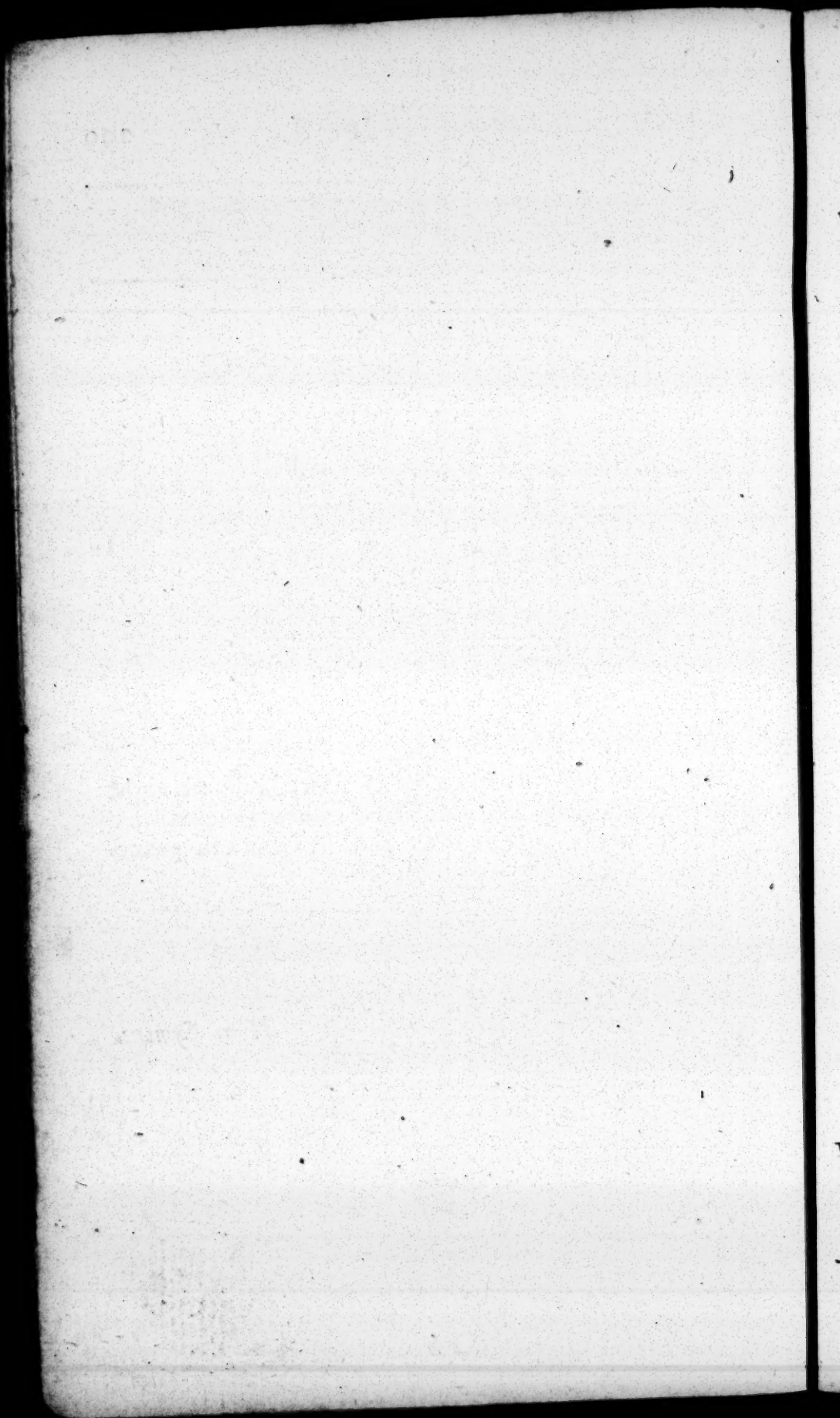
Voucher from 9ber 10th, to 10ber 22d, 1768.

	Weight.	L.	s.	d.
Column	4894	20	7	10

The amount of this Voucher is four thousand eight hundred and ninety-four pounds of tallow candles.

Cash is twenty pounds seven shillings and ten pence.

Wm. Symons.



S O P E.

_____ Collection.

_____ Division.

Fourth Round Voucher.

From 9ber 10th, to 10ber 22nd, 1768.

Weight.

14784 at $1\frac{1}{2}$ per pound

L.	s.	d.
92	8	0

CASH, ninety-two pounds and eight shillings.

APPENDIX.

303

Sope Voucher from 9ber 10th, to 10ber 22nd, 1768.

	Net weight.	L.	s.	d.
Column	14784	92	8	0

The amount of this Voucher is fourteen thousand seven hundred and eighty-four pounds of sope.

Cash is ninety-two pounds and eight shillings.

Wm. Symons.

P

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F
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B
D
V
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C

P A P E R.

_____ Collection.

_____ Division.

Fourth Round Voucher.

From 9ber 10th, to 10ber 22nd, 1768.

Specie.	N ^o	at what	Value.			Duty.		
			L.	s.	d.	L.	s.	d.
Fools cap,	29	18d.	—	—	—	2	3	6
Ditto second,	49	13½	—	—	—	2	15	1½
Fine pot,	170	18	—	—	—	12	15	0
Second ditto,	250	9	—	—	—	9	7	6
Brown large cap,	105	9	—	—	—	3	18	9
Ditto small ordinary,	88	6	—	—	—	2	4	0
Whited-brown,	68	9 per bundle	—	—	—	2	11	0
Charged ad valorem	240	18l. per cent.	20	1	2	3	12	2¾
Total —						39	7	1¼

CASH, thirty-nine pounds seven shillings and one penny farthing.

<i>Collection.</i>		<i>Division. Fourth Round</i>						
	Date of last charge	Fools cap fine at 1s. 6d. per ream.	Fools cap second at 1s. 1½d. per ream.	Fine pot at 1s. 6d. per ream.	Second pot at 9d. per ream.	Brown large cap, at 9d. per ream.	Small ord. Brown at 6d. per ream.	Whited - brown at 9d. per bundle.
John Crawley,	Dec. 18.	29	49	170	250	—	—	—
George Green,	Ditto 17.	—	—	—	—	105	88	68
Total		29	49	170	250	105	88	68

Paper Voucher from 9ber 10th, to 10ber 22d, 1768.

Ordinary cap.	Bluegrey.	Sky blue.	No. of reams charg- ed ad valorem.	Value at 18l. per cent.			Duty.		
valued at s. d. s. d. s. d.				L.	s.	d.	L.	s.	d.
33	16	15							
28	42	—	7	7	14	0	28	8	10 $\frac{1}{4}$
—	76	94	170	12	7	2	10	18	3
28	118	94	240	20	1	2	39	7	1 $\frac{1}{4}$

The amount of this Voucher is twenty-nine reams of fools-cap fine, forty-nine ditto of second, one hundred and seventy reams of fine pot, two hundred and fifty ditto of second, one hundred and five reams of large brown cap, eighty-eight reams of small ordinary brown, sixty-eight bundles of whited-brown, with two hundred and forty reams charged ad valorem, valued at twenty pounds one shilling and two-pence.

Cash, thirty-nine pounds seven shillings and one penny farthing.

Wm. Symons.



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H I D E S.

Collection.

Division.

Fourth Round Voucher.

From Nov. 10, to Dec. 22, 1768.

Specie.	N ^o	Wt.		<i>l.</i>	<i>s.</i>	<i>d.</i>
Hides, &c.	638	1912	at $1\frac{1}{2}d.$ per Pound	11	19	0
	<i>l. s. d.</i>					
Pieces, &c.	930	168	at 30 <i>l.</i> per Cent.	0	5	0
			Tanners	12	4	0
Sheep, &c.	268	335	at $1\frac{1}{4}d.$ per Pound	1	14	$10\frac{3}{4}$
	D. O					
Slink without	7:8		at 1 <i>s.</i> per Doz.	0	7	8
Ditto with	7:8		at 3 <i>s.</i> per Doz.	1	3	0
Horie hides	231		at 1 <i>s.</i> 6 <i>d.</i> per Hide	17	6	6
	<i>l. s. d.</i>					
Pieces, &c.	200	76	at 30 per Cent.	0	2	3
			Tawers	20	14	$3\frac{3}{4}$
Sheep, &c.	2000	2392	at 3 <i>d.</i> per Pound	29	18	0
Deer, &c.	464	522	at 6 <i>d.</i> per Pound	13	1	0
Pieces, &c.	6	3	at 15 <i>l.</i> per Cent. and 2 <i>d.</i> per pound	0	1	3
			Oil Dreffers	43	0	3
			Total	75	18	$6\frac{3}{4}$

CASH is seventy-five pounds eighteen shillings and six pence three farthings.

Collection. Division. Fourth

	Date of last Change.	Hides and Skins at 1½ per lb.		Sheep and Lamb at 1¼ per lb.		Calves, &c. without Hair, at 1s. per Doz.		Beaver and Goat, at 1s. per Dozen.		Calves with hair on, at 3s. per Doz.		Horses Hides, at 1s. and 6d.	
		No.	Wt.	No.	Wt.	D.	O.	D.	O.	D.	O.	No.	
John Smith	Dec. 18th	443	1302										
John Jones	16	195	610										
Tanners Total		638	1912										
Thomas Lane	19th			245	300	4	4			5	2	214	
Henry Wood	18			23	35	3	4			2	6	17	
Tawers Total				268	335	7	8			7	8	231	
John Jones	17th												
Henry Rogers	17												
Oil Dr. Total													

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No

600
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2000

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APPENDIX.

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Round Hide Voucher from Nov. 10, to Dec. 22, 1768.

Sheep and Lamb, at 3d. per lb.		Deer and Goat, at 6d. per lb.		Pieces ad valorem.	Value at 30l. per Cent.			Pieces ad valorem, at 2d. per pound.	Value at 15l. per Cent.			Duty.		
No.	Wt.	No.	Wt.	N.	L.	s.	d.	n. w.	L.	s.	d.	L.	s.	d.
				80		13	4					8	6	9
				13		3	4					3	17	3
				93		16	8					12	4	0
				14		14	7					18	13	5 $\frac{1}{2}$
				6		2	11					2	0	10 $\frac{1}{4}$
				20		17	6					20	14	3 $\frac{3}{4}$
600	452	240	270					2	1	1	8	12	8	5
1400	1940	224	252					4	2	3	4	30	11	10
2000	2392	464	522					6	3	5		43	0	3
Total												75	18	6 $\frac{3}{4}$

Tanned, { The amount of this voucher is fix hundred thirty-eight hides and skins, weighing one thousand nine hundred and twelve pounds; with ninety-three pieces ad valorem, value sixteen shillings and eight pence.

Tawed, { Two hundred sixty-eight sheep, &c. weighing three hundred thirty-five pounds; seven dozen and eight skins flink, without hair; seven dozen and eight skins with hair on; two hundred thirty-one horse hides, with twenty pieces ad valorem, value seventeen shillings and six pence.

Dressed in oil { Two thousand sheep, &c. weighing two thousand three hundred and ninety-two pounds; four hundred sixty-four deer, &c. weighing five hundred and twenty-two pounds, with six pieces ad valorem, weighing three pounds, value five shillings.

Cash, seventy-five pounds eighteen shillings and six pence three farthings

Wm. Symons.

1768.

Division.

Fourth Round.

General Abstract.

Compare	Receipts Arrears	Total	Arrears Amount	Excise.	Cyder 1766.		Malt.						Candles.	Sope.		Paper.		Hides.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
					L.	s.	Determined.	Growing.		Additional.	L.	s.		L.	s.	L.	s.		Paper.	L.	d.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
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Division 4th Round

	C.brewers.		Vituallers.			Duty.			Malt duty.	No. of hhd.
	x.	vi.	x.	vi.	Cyder.	L.	s.	d.		
James Humphrey	120	78 $\frac{3}{4}$	—	—	—	47	9	3	thil.s	—
Daniel Wright	—	—	20 $\frac{1}{2}$	16	1 $\frac{1}{2}$	9	15	4	6	1 $\frac{1}{2}$
George Gale	—	—	18 $\frac{1}{4}$	10 $\frac{1}{4}$	4 $\frac{1}{2}$	9	10	—	18	4 $\frac{1}{2}$
Anne Leach	—	—	30 $\frac{1}{2}$	18	3	14	8	—	12	3
Thomas Cart	—	—	28 $\frac{1}{2}$	20 $\frac{1}{2}$	4	13	18	—	16	4
John Jones	—	—	14 $\frac{1}{2}$	10	2 $\frac{1}{2}$	7	4	—	10	2 $\frac{1}{2}$
William Stokes	—	—	10 $\frac{1}{2}$	9	2 $\frac{1}{4}$	5	13	—	9	2 $\frac{1}{4}$
By Ales	—	—	3 $\frac{3}{4}$	4 $\frac{1}{2}$	—	1	16	—	—	—
Excise	120	78 $\frac{3}{4}$	126 $\frac{1}{2}$	88 $\frac{1}{2}$	17 $\frac{3}{4}$	109	13	7	10t.	17 $\frac{3}{4}$

	Net wt. of sope.	Duty.		
		L.	s.	d.
James King	10991	68	13	10 $\frac{1}{2}$
John Hall	379	23	14	1 $\frac{1}{2}$
Sope.	14784	92	8	0

	Fools cap fine at 1s. 6d. per ream.	Fools cap second at 1s. 1d. per ream.	Fine pot at 1s. 6d. per ream.	Second pot at 9d. per ream.	Brown large cap at 9d. per ream.				
John Crawley	29	49	170	250	—				
George Green	—	—	—	—	105				
Paper	29	49	170	250	105				

	Hides and skins at 1d. per pound.	Sheep, &c. at 1d per pound.	Calves without hair, at 1s. per dozen.	Calves with hair, 3s. per dozen.	Horse hides, at 1s. 6d.	Sheep, &c. at 3d. per pound.
	Wt.	Wt.	D.O.	D.O.		Wt.
John Smith	1302	—	—	—	—	—
John Jones	610	—	—	—	—	—
Tanners Total	1912	—	—	—	—	—
Thomas Lane	—	300	4	5	2	214
Henry Wood	—	35	3	4	2	6
Tawers Total	—	335	7	8	7	8
John Jones	—	—	—	—	—	—
Henry Rogers	—	—	—	—	—	—
Oil dressers tot.	—	—	—	—	—	—
						452
						1940
						2392

General Abstract

Cyder 1766			Belt.	Floor.	Cyder.	Old duty			Add duty		
L.	s.	d.				L.	s.	d.	L.	s.	d.
—	9	—	308	161	—	8	3	5 $\frac{1}{2}$	4	1	8 $\frac{3}{4}$
1	7	—	312	163	—	8	5	6 $\frac{1}{2}$	4	2	9 $\frac{3}{4}$
—	18	—	310	—	—	6	4	—	3	2	—
1	4	—	—	—	1	—	4	—	—	—	—
—	15	—	—	—	17 $\frac{3}{4}$	3	11	—	—	—	—
—	13	6	—	—	—	—	—	—	—	—	—
—	—	—	930	324	18 $\frac{3}{4}$	26	8	—	11	6	6
5	6	6	Malt.								

Charles Wheeler
James King
Candles

Wt.	Duty.		
	L.	s.	d.
1820	7	11	8
3074	12	16	2
4894	10	7	10

								Small ordinary brown at 6d. per ream.				Whited brown at 9d. per bundle.				No. of reams charged ad valorem.				Value at 18l. per cent.				Duty.																			
								L.				s.				d.				L.				s.				d.															
								—				—				70				7				14				0				28				8				10 $\frac{1}{4}$			
								88				68				170				12				7				2				10				18				3			
								88				68				240				20				1				2				39				7				14 $\frac{1}{4}$			

				Deer, &c. at 6d. per pound.				Pieces ad valorem.				Value at 30 per cent.				Pieces ad valorem at 2d. per pound.				Value at 15 per cent.				Duty.																																			
				Wt.				L.				s.				d.				Duty.				Wt.				L.				s.				d.																							
				—				80				—				13				4				—				—				—				—				8				6				9											
				—				12				—				2				4				—				—				—				—				2				17				3											
				—				93				—				16				8				—				—				—				—				12				4				0											
				—				14				—				4				7				—				—				—				—				18				13				5 $\frac{1}{2}$											
				—				6				—				2				11				—				—				—				2				0				10 $\frac{1}{2}$															
				—				20				—				7				0				—				—				—				20				14				3 $\frac{1}{2}$															
				270				—				—				—				—				2				1				—				—				1				8				12				8				5			
				250				—				—				—				—				4				2				—				—				3				4				30				11				10			
				522				—				—				—				—				6				3				—				—				5				—				43				0				3			

Win. Symons.

Total 175 18 6 $\frac{1}{2}$

7th Month.

Collection.

Division.

Distillery Voucher.

From 10ber 31, to Jan. 31, 1768.

Gallons of		Specie.	Low wines			Spirits.		
L. wine	Spirits.		L.	s.	d.	L.	s.	d.
5000	3000	Malted corn	208	6	8	375	—	—
8200	4100	Cyder	478	6	8	456	19	7
1920	1280	F. wine, &c.	264	—	—	133	6	8
Total.			950	13	4	965	6	3

CASH for low wines, is nine hundred fifty pounds thirteen shillings and four pence.

Ditto for spirits, nine hundred sixty-five pounds six shillings and three pence.

<i>Collection.</i>				<i>Division. Distillery</i>			
	Date.	L. wines From malted corn &c. Gallons.	Spirits Gal's.	L. wines From Cyder Galls.	Spirits Galls.	L. wines From wines &c imported. Galls.	Spirits Galls.
William Smith	Jan.						
	6	1005	640	—	—	—	—
	7	995	550	—	—	—	—
	9	1050	620	—	—	—	—
	12	1000	610	—	—	—	—
	14	950	580	—	—	—	—
	—	5000	3000	—	—	—	—
	18	—	—	2050	1005	—	—
	20	—	—	1945	950	—	—
	22	—	—	2055	1050	—	—
James Wintle	24	—	—	1950	995	—	—
	—	—	—	8000	4000	—	—
	20	—	—	105	52	—	—
	24	—	—	95	48	—	—
John Allen	—	—	—	200	100	—	—
	20	—	—	—	—	520	320
	24	—	—	—	—	490	300
	26	—	—	—	—	500	340
	30	—	—	—	—	410	320
Thomas Millar	—	—	—	—	—	1920	1280
		5000	3000	8200	4100	1920	1280

Voucher from 10ber 31st to January 31st, 1768.

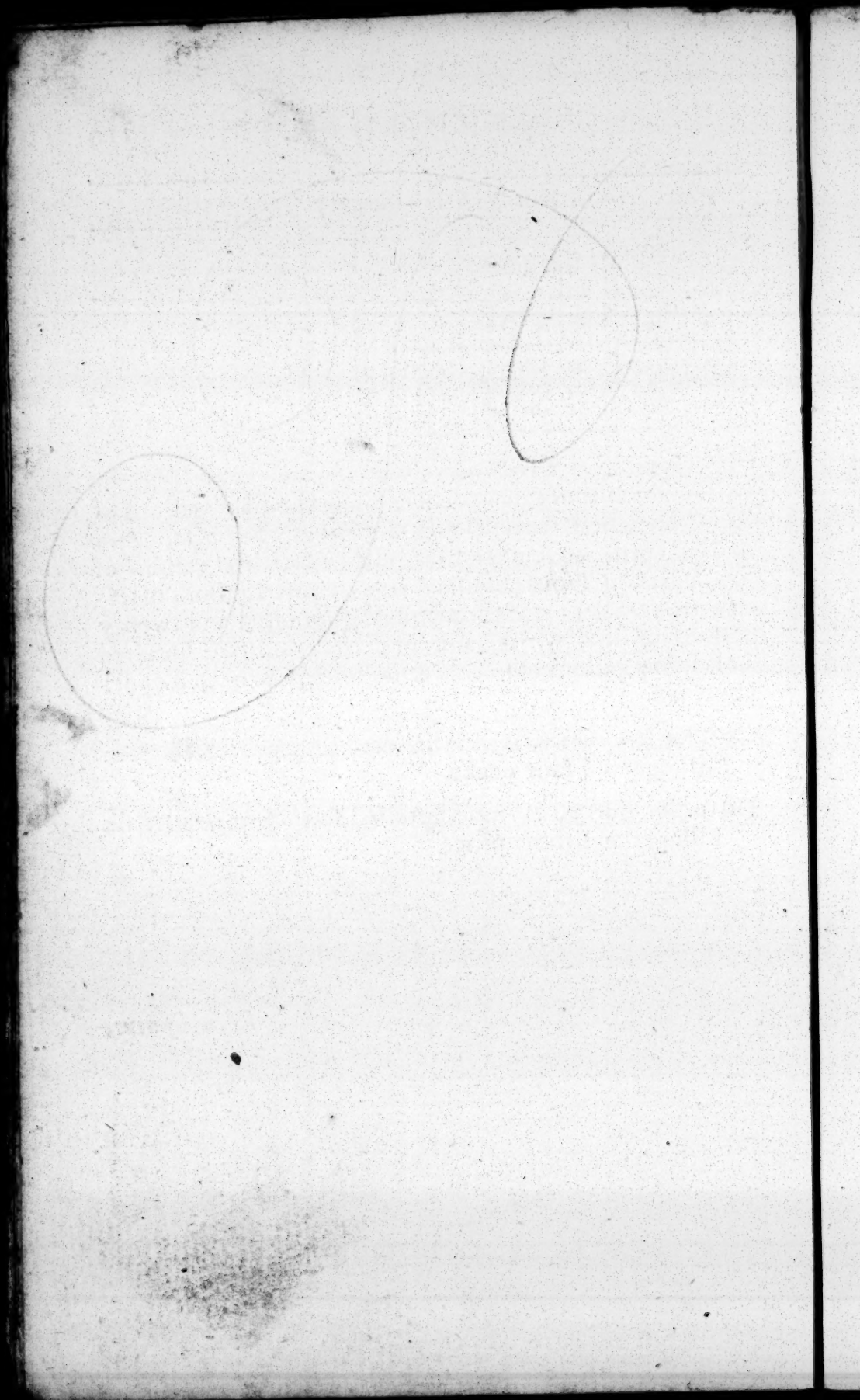
	Gallons of		Low wines			Spirits.		
	L.W	Spit.	L.	s.	d.	L.	s.	d.
1 Column	5000	3000	478	6	8	456	19	7
2 Column	8200	4100	208	6	8	375	0	0
3 Column	1920	1280	264	0	0	133	6	8
Total			950	13	4	965	6	3

The amount of this voucher, is five thousand gallons of low wines, and three thousand gallons of spirits, from malted corn, &c. eight thousand two hundred gallons of low wines, and four thousand one hundred gallons of spirits, from cyder; one thousand nine hundred and twenty gallons of low wines, and one thousand two hundred and eighty gallons of spirits, from foreign wine and cyder imported.

Cash for low wines, is nine hundred fifty pounds thirteen shillings and four pence.

Ditto for spirits, is nine hundred sixty-five pounds six shillings and three pence.

Wm. Symons.



— Division Diffillery Abstract for 7th Month 1768.

From what species.	Areas.			Amount.			Receipt.			Arrears depending		
	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.
	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.
Malted corn, &c.	20	16	8	37	10	—	—	—	—	—	—	—
Cyder.	46	13	4	44	11	8	—	—	—	—	—	—
Foreign wine, &c.	26	8	—	13	6	8	—	—	—	—	—	—
Total	93	18	—	95	8	4	—	—	—	—	—	—

Division Distillery Abstract

	L. wines	Spirits.
	Gallons.	Gallons.
William Smith	5000	3000
James Wintle	8000	4000
John Allen	200	100
Thomas Millar	1920	1280

APPENDIX.

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from 10 ber 31st to January 31st, 1768.

Species.	Low wines			Spirits.		
	L.	s.	d.	L.	s.	d.
From malted corn, &c.	208	6	8	375	0	0
From cyder	466	13	4	445	16	8
From ditto	11	13	4	11	2	11
From foreign wine, &c. imported	264	0	0	133	6	8
Total	950	13	4	965	6	3

Wm Symons.

F I N I S.

This day is published,

Price Four Shillings and Sixpence bound,

The SECOND EDITION,

Greatly enlarged and improved, delicated (by permission) to the ATTORNEY GENERAL,

THE EXCISE LAWS abridged and digested under their proper heads, in alphabetical order; a work designed not only for the use of the officers, but for every person liable to the excise duties, or subject to the survey of the excise officers, shewing them what duties they are respectively liable to, what regulations and restrictions they are under, and what penalties they incur by transgressing them; and intended also for the use of Justices of the Peace, and the gentlemen of the law, as an index to those laws in the statutes at large.

By JELINGER SYMONS.

Printed for F. WINGRAVE, in the Strand.



